# Considerations, design calculations, construction, test and evaluation of a cavity walled solar oven.



By

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#### Contents.

- 1. Abstract.
- 2. The background.
- 3. Ladakhi solar-ovens.
- 4. Svante Arrhenius' law.
- 5. A description of the first proto type.
  - 5.1. The flammability of wood.
- 6. Required operating temperatures for solar ovens related to cooking and health issues.
- 7. The solar irradiation.
  - 7.1 Tilting the oven to face the sun.
  - 7.2 The horizontal alignment of the tray.
- 8. Thermal properties.
  - 8.1 The thermal capacitance.
  - 8.2 The transient temperature response and the thermal time constant.
  - 8.3 The thermal resistance.
    - 8.3.1 The thermal conduction.
    - 8.3.2 The convection.
    - 8.3.3 The radiation.
  - 8.4 Configuration and calculations for a thermal equivalent of a solar oven.
    - 8.4.1 Power accounts and Kirchhoff's law.
    - 8.4.2 The diagram of the thermal model of the solar oven.
    - 8.4.3 The power (i.e. heat) irradiated into the oven.
  - 8.5. Calculating the thermal resistances using the temperatures and the heat flow contributions.
    - 8.5.1 Check and balance in the account of: heat flows, thermal resistances and temperatures.
- 9. Issues of measuring the thermal properties of a solar oven.
  - 9.1 On the properties of solar ovens. By Torkil Heiede and Gunnar Bomann.
  - 9.2. My own unsuccessful attempt to deduce a general analytic expression for P x  $R_{th}$  and  $\tau$ .
  - 9.3. The un-linearity of the thermal resistances as a function of the temperature.
- 10. Design considerations.
  - 10.1. The thermal trade-offs of the size and shape of the solar oven.
  - 10.2. Insulation of the air gap in the solar oven.
  - 10.3. Insulation materials.
- 11. Dressing up the solar oven in coat and skirt.
  - 11.1. The impact of a coat on the temperature of the oven.
- 12. Pots and pans used in the East African kitchen.
- 13. Using the solar oven as a hay box
- 14. Notes, mistakes made, lessons learned, recommendations, tasks and rectifications.
- 15. Other cooking technologies.
- 16. Thanks given.
- 17. Conclusion.
- 18. Appendix; comprising test results and plots of temperature measurements.

**1. Abstract.** The use of fossil fuels presents many problems, not least in refugee camps. Single walled solar ovens have proved to work in Ladakh (in the Himalayas), although, a higher operating temperature is desired. This report investigates the feasibility of designing a dual (i.e. cavity) wall and doubled glazed solar oven, with the aim to raise the operating temperature to as near to 100°C as possible. The present design has aimed to comply with the living conditions in - and to use cheap materials and tools typically available in- refugee camps. Simple technology and low cost have also been priorities.

A prototype has been built and tested in Aarhus Denmark (Latitude 56° 16'N; Longitude 10° 20'E), where it reached a maximum temperature of 83.0°C on August 13<sup>th</sup> 2022. The test of the oven complied well with the theoretical calculations. The obtained results indicate good chances for further improvements, such as to raise the temperature by 5–15°C – or more - allowing for low pasteurizing of food and liquids, which requires 72°C for 15–20 seconds; or high pasteurizing, 87°C for 15–20 seconds.

The conclusion includes the "lessons learned", suggestions for tasks and improvements to be materialized in a second "Mark two" prototype for a further increase of its operating temperature.

The test of the solar oven also suggests to use the oven to produce sterilized water / drinkable liquids. and for possible preservation of fruit and vegetables (in jars). Many (if not most) villages in South Western Zambia lack access to clean drinking water, and muddy drinking water has brought the average lifetime in the area down to 29 years and the infant mortality up to 60%.

In clouded periods, the solar oven can also be used as a hay box. Hay boxes have been used in Europe for more than a century, and for the last 50 years I have cooked using my bed and a towel as a hay box. Hay boxes are also called: fireless cooker, Norwegian cooker, retained-heat cooker, insulation cooker, straw box, wonder oven or self-cooking apparatus. A good hay box will save a significant amount of fossil fuel (I guess between 30 - 70%) - at the expense of a prolonged cooking time – depending on the meal and the amount of food / liquid to be processed.

The appendix comprises the thermal measurements and the associated graphs for the oven..

This report is aimed at refugees, refugee camp managers, NGOs and other organisations, do it yourself people – and who ever might have an interest. Collaboration with- and feed-backs from readers are most welcomed and will be much appreciated.

### 2. The background.

My name is Steen Carlsen. I was born in 1951. My background is a M.Sc. in electronics and electric engineering. I have lectured in thermal design at the University of Aarhus, Denmark. I have travelled extensively in Asia and East Africa (including Zambia, Uganda etc.).

A friend of mine - Stig - has formerly worked for the Danish Refugee Council. Stig has drawn my attention to the lack of firewood in many refugee camps, where firewood is generally the only heating source available for cooking. Often the cooking is done in a pot balancing over an open fire on top of three stones. For a pot on top of an open fire the efficiency (i.e., the fraction of energy transferred from the firewood to the food - divide by the amount of energy in the fire) is generally in the order of 2%. It means that 98% of the energy is wasted.

The consumption of firewood (and whatever can burn – including dung, which is essential for fertilization) – frequently leads to deforestation and desertification. When Stig told me about the situation in the refugee camps, it reminded me about the solar-ovens, which I saw in the summer of 1991, while I worked as volunteer with Ladakh Ecological Development Group, in the city of Leh, (altitude = 3500 m) in Ladakh in the Himalayas (~ 400 km east of the Kashmir Valley).

I suggested to Stig to design and build a prototype of a solar oven with an improved insulation (to obtain a higher operating temperature) and - to the widest possible extent – based on tools and materials, which we believe to be available in the refugee camps. Stig agreed. My friend Thomas arranged for access to a well-equipped workshop and he has contributed with numerous work hours.

### 3. Ladakhi solar ovens.

The solar ovens I saw in Leh in Ladakh, were made from a few wooden boards. Inside, the ovens were painted black. They had triangular ends, and their rear sides were mounted perpendicular to the bottom. Generally, a pane / an old window in a wooden frame - hinged to the top of the rear side, played the role as an inclined top / front side. This front side / window could be opened, when dough for bread and cakes were placed in the oven for baking, and the pane / glass was turned to face the sun. During that summer, I enjoyed numerous delicious cakes baked in the solar ovens.

I got the impression that the baking time for bread in Leh was in the order of 8 hours. I have used that baking time to estimate the temperature in the Ladakhi solar ovens. The baking time in a normal electrical oven (at sea level) is in order of one hour. For as long as there is moisture left in the bread (or the dough) the temperature inside a bread will not exceed the boiling point (at sea level =  $100^{\circ}$ C).

This is because - at the boiling point - all heat applied to the bread will go to convert the moisture / liquid inside the bread - into vapour. Thus, even if we adjust the temperature of a grid-connected electric oven to e.g. 180°C, the temperature within the bread will not exceed the boiling point. The boiling point (100°C at sea level) decreases by 0.5°C by every 152.4 meters (= 500 feet) increase in altitude. Hence, the boiling temperature in Leh at an altitude of 3500 m is approximate 88°C.

#### 4. Svante Arrhenius' law.

To get an idea of the operating temperature of the solar ovens in Ladakh, I applied a rule of thumb - derived from Svante Arrhenius' law – which states that the speed of a chemical process increases by a factor (Q) = 2.3 times by every 10 degrees increase of the temperature of the process. E.g., by a temperature increase of 30 degrees the reaction speed will rise by 2.3 x 2.3 x 2.3 =  $2.3^3 = 12.17$  times. Opposite, if the temperature drops 30 degrees, the speed of the process will drop  $2.3^{-1}$  x  $2.3^{-1}$  x  $2.3^{-1} = 2.3^{-3} = 1/(12.17)$  times = 8.2% of the initial speed.

In section 3, I described the Ladakhi solar ovens. I mentioned, that no matter the temperature of the oven, the highest possible temperature inside the oven would not exceed  $100^{\circ}$ C. For an oven with a temperature lower than  $100^{\circ}$ C, the temperatures of the oven and of the dough are more or less the same. I also mentioned that for an oven at  $180^{\circ}$ C the baking time  $t_1$  is in the order of one hour.

Now, let us start with an oven, which is  $100^{\circ}$ C. I want to find the temperature – expressed by a number K times ten degrees – by which I should lower the temperature of the oven, if I wanted a baking time  $t_2$ , which would be 8 hours (i.e.  $t_2/t_1 = 8$ ).

That means  $t_2$  is 8 times longer than the 1.0 Hour baking time ( $t_1 = 1.0$  Hour for an oven at  $100^{\circ}$ C).

- (1)  $Q^K = t_2/t_1 =>$
- (2) K log Q = log  $t_2/t_1 = >$
- (3)  $K = (\log t_2 / t_1) / \log Q$
- $(4) = \log 8 / \log 2.3$

 $= 2.49 \text{ (times } 10^{\circ}\text{C)}.$ 

Hence, I assume that the temperature in the solar ovens in Ladakh was in the order of

$$T = 100^{\circ}C - 2.49 \times 10^{\circ}C \approx 75^{\circ}C.$$

# 5. A description of the first prototype.

This solar oven is dual (or cavity) walled. I.e. it consists of two boxes, one inside the other and a spacing of 10 mm between the bottom and the sides of the inner- and the outer box. The inner box is made out of 10 mm chipboard / particleboard – the outer box is made of 15 mm chipboard (see fig. 2).

A 0.01 mm thick aluminium foil is glued on to the vertical sides between the two boxes and to the outer bottom side of the inner box. See fig.3). As can be seen at fig. 3 next page, the



outside of the bottom of the inner box is raised 10 mm

Fig. 2. above the inside of the bottom of the outer box, to allow for insulation between the two boxes. To reflect the sun, aluminium foil is glued on to the inner vertical sides of the inner box.

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To limit the heat *conduction* from the inner- to the outer box, the only thermal connection between the two boxes are the tiny ( $\emptyset \approx 4$  mm) heads of the four small shiny screws in the bottom of the inner box, (seen at the corners of the box at fig. 3). The heads of the four screws carry the weight of the inner box. Their heads rest at the top side of the bottom of the outer box.

Also, at the top side of the bottom of the outer box (fig. 4), you find eight alignment blocks, which allow for the inner box to be symmetrically positioned in the outer box. This is in order to ensure:

A. An easy assembly and dismantling of the inner box - and B. To secure an evenly distributed spacing for the insulation around the inner box.

The measurements of the interior of the inner box are Height: 210 mm; Width: 300 mm; Length: 490 mm.

The measurements of the outside of the outer box are Height: 280 mm; Width: 380 mm; Length: 575 mm.

The weight of the entire oven is 18 kg.

The British encyclopaedia Britannica is an excellent source to properties of wood; its Hyperlink / Link is:



Fig. 3.



Fig. 4.

https://www.britannica.com/science/wood-plant-tissue/Thermal-properties.

The density of the chipboard used for this oven is 635 kg/m³, whereas the density of plain mature (>20 years) woods such as pine-, spruce- and fir- woods range in the order of 300 - 500 kg/m³. On top of the lower density comes that both plain- and plywood have a higher tensile strength than chipboard, thus, allowing for less material = thinner plates. I expect plain wood or plywood to cut the weight by 30-60%.

The thermal conductivity of wood perpendicular to the grain is generally within the range of 0.1-0.2 W/mK. The thermal conductivity is Fig. 5. highest in the axial direction and increases with density and moisture content. Thus, light dry woods are better insulators. The thermal conductivity of chipboard is typical 0.15 W/mK. For plywood it ranges from 0.11 to 0.15; average is 0.13 W/mK. The thermal conductivity of glass is ~1 W/(mK).

The thermal expansion coefficients are for: glass 6.7  $10^{-7}$  K<sup>-1</sup>; & wood parallel to grain:  $30 \times 10^{-7}$  K<sup>-1</sup>. The length L of the glass = 0.53 m. Increasing the temperature from 0 - 100 °C = 100 K; causes the length of the glass to exceed the length of the wood in the frame by:  $\Delta L = 100$  K \* 0.53 m \* (67 - 30) x  $10^{-7}$  K<sup>-1</sup> = 0.196 mm. I suggest a play / clearance of ~1.0 mm.

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The thermal density of dry wood ranges from 4100 - 6800 kilocalories per kg., with an average of 4500 kilocalories per kg. (Britannica). 1.0 J = 0.239 Calorie.

The heat capacity for Softwood Plywood (bulk density  $450-500 \text{ kg/m}^3$ ) & temperatures  $<100^{\circ}\text{C}$  is: Heat capacity =  $0.75 \text{ MJ} / (\text{m}^3 \, ^{\circ}\text{C})$ . By temperatures  $100-200^{\circ}\text{C}$ : Heat capacity =  $2.4 \text{ MJ} / (\text{m}^3 \, ^{\circ}\text{C})$ . (From Britannica "Loadbearing capacity of cold formed steel joists subjected to severe heating.")

Fig 5. shows the screws through the two handles, which can be used to lock the two boxes of the oven, the glass lid and the protective lid together into one unit, during transport. The purpose of the long screws through the holes in the handles is also to secure and protect the pane by covering it with the protective lid (also chipboard) and clamping the later to the outer box by fastening the screws into the outer box.



Fig. 6.

Fig. 6. shows how the glazed lid is (similar to a pane) a mortised frame made out of plain lumber and equipped with double (top 5.0 mm / bottom 4.0 mm) standard window glass.

The distance between the two panes is 22 mm. The total thickness of the frame of the lid is 40 mm. The lid, however, only extends 20 mm above the upper edge of the outer box. That is because the lower pane rests on top of the edge of the inner box.

As the window lid adds 20 mm to the total height, it brings the total height of the oven up to 300 mm. Mounting the protective (chipboard) lid on top further adds yet another 15 mm to the height, bringing the total height of the oven up to 315 mm.

Fig. 7. shows two black wooden blocks (barely to be seen) in diametrically opposite corners of the inner box. Their purpose is to act as handles when the inner box must be lifted out of the outer box. The same picture also shows the 24V fan (used to circulate the air during the tests when the oven was electrically heated) and the steel tray, which carried the cooking plate.



Fig. 7.

**5.1 The flammability of wood.** Wood must be raised to a temperature of approximately 250°C for a spark or a flame to ignite it. At approximate 500°C the ignition occurs spontaneously.

Hence, if / when the oven is being used as a hay box, hot pots and pans - when shifted from a fire place or stove to the bottom of the oven - could - if worst comes to worst - set the oven on fire. Thus, measures must be taken to prevent the bottom of the oven to be expose to excessive temperatures. One measure could be a sheet of metal (e.g. steel, aluminium copper) placed on top of a few pebbles or some tiny pieces of clay. The most suitable metal is eloxal (black) aluminium, copper is also great and an (even rusty) iron sheet will do.

# 6. The required operating temperatures of a solar-oven - related to cooking and health issues.

On August 14<sup>th.</sup> 2022 I met a student on a bus ride. He studied for his MSc. in food science and technology at the Danish Technical University. He and my friend Henrik, a medical practitioner, have both provided valuable information on various specific temperatures in food processing. Starting from the low temperatures, here are some milestones in heating of food:

63°C / 72°C: Low pasteurization includes heating of milk to 63°C for a duration of 30 minutes - or at 72°C for 15 seconds duration - and then immediate cooling (less than 3°C). This process kills all pathogenic bacteria and reduces the load of spoilage bacteria, with no impact on the taste. Some denaturation of proteins (3%–5%) can be expected. (I have, for decades preserved fruit using 72°C for 15 seconds and obtained good shelf lives beyond four years.)

82°C: To boil a perfect soft egg, the yolk must remain below 70°C, while the egg white must reach a temperature beyond 82°C.

87°C / 90°C: High pasteurization requires a heating to 87°C for 15 seconds - or 90°C for 5 seconds. Opposite to low pasteurization - high pasteurization can have an impact on the taste of the food.

80°C / 100°C: To quote the Danish institute "Statens Serum Institut": "Clostridium botulinum is a common bacterium in nature, which produces some of the most lethal and dangerous neuro toxins known. The botulinum toxin can however, be inactivated by sunlight (1-3 hours), high temperatures (80°C for 30 minutes or 100°C for a few minutes) or hypochlorite (0.5%)".

Due to: **A**. The issues mentioned above – and **B**. The wish for fast cooking

- I will advocate to strive to attain a temperature – as near to water's boiling point as possible – without causing harm to the materials of the oven.

Some studies of solar pasteurization have focused on water pasteurization, knowing that diseases caused by water are the reason for five million deaths per year (Burch & Thomas, 1998). In South West Zambia, many villages have a life expectancy of 29 years and an infant mortality around 60% due to muddy drinking water, which causes wide spread diarrhoea.

#### 7. The solar irradiation.

Outside the atmosphere of the Earth, the solar irradiation is  $\sim 1300 \text{W/m}^2$ . In Denmark (at sea level and a latitude of  $56^0\text{N}$ ), the maximum irradiation is generally rated as  $1000 \text{ W/m}^2$ . For Central and East Africa, I have assumed a maximum irradiation to be between these two values – presumably only slightly above  $1000 \text{ W/m}^2$  - on a cloudless day.

Yet, for the last 5 years, I have monitored the current produced by my solar panels. I have noticed that whenever a cloud shadows for the sun, the current, produced by the panels, drops to approximate 5% of the maximum current (i.e. by unclouded skies). I do not know if the irradiation of heat to a solar oven will exhibit the same susceptibility to clouds as the electric solar panels. I suppose it does.

For a solar oven, which has no mirrors or lenses, there will be no means to increase the intensity I  $[W/m^2]$  of the sun. Thus, by unclouded skies the maximum heating power we can extract from the sun will be:

$$P = I A \cos \theta$$
 - where

I  $[W/m^2]$  is the intensity of the sun.

A [m<sup>2</sup>] is the area of the glass in a solar oven, which is facing the sun.

 $\theta$  [in angular degrees] is the angle between a line to the sun and a line perpendicular to the pane.

One of the disadvantages of solar ovens is their - over the day - regularly need for realignment, such that the sun beams irradiate / enter perpendicular ( $\theta = 90^{\circ}$ ) into the pane. That means, that the irradiation angle must be perpendicular to the glass in both the vertical and in the horizontal plane. Hence regular adjustments are required both horizontally and vertically. Any deviation from  $\theta = 90^{\circ}$  will reduce the temperature of the oven.

Fortunately, the  $\cos\theta$  factor in the equation P=I A  $\cos\theta$  causes the power reduction to be insignificant for minor deviations. The graph in fig. 8 shows how many percent of the maximum obtainable power you will lose for a given angular deviation from a perpendicular line of incidence. At the graph, I have plotted the  $\cos\theta$  function and underneath the associated angles are shown.

For a 10° deviation from perpendicular you will suffer a 2% loss in the solar power. By 20° deviation the loss will be 6%. Unless the oven has an exceptionally good insulation, (and hence, a high operating temperature), a 6% loss of power might endanger the lower limit of 72°C, required for a safe cooking.

Thus, I will not recommend a deviation of more than  $15^{\circ}$  from perpendicular – corresponding to ~4% loss of power. If we assume A. The sun to describe an arc of  $180^{\circ}$  over a day of 12 hours, and B. That each adjustment of the position of the solar oven brings it (back) to the exact alignment, then  $180^{\circ}/15^{\circ} = 12$  daily adjustments are needed, to maintain a power loss less than 4%.

If, however, by each adjustment of the oven's position will be shifted from 15° behind the trajectory of the sun - to 15° ahead of the sun, the same power loss can be obtained for only 6 daily adjustments. Yet, this requires a deeper understanding by the person, who is responsible for the alignment.

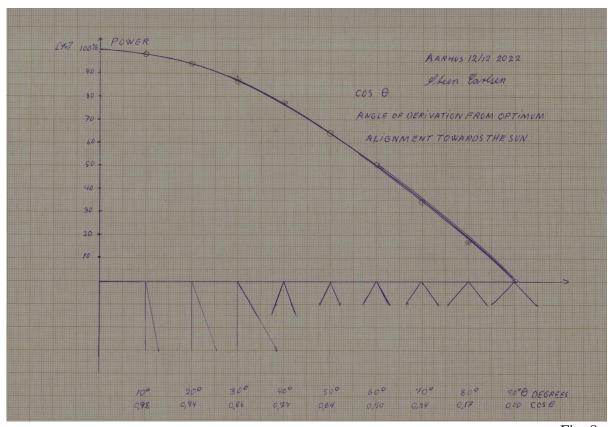


Fig. 8.

The gadget shown below on fig. 9 is meant to ease the process, when the oven must be inclined and aligned to face the sun. The idea is to mount an approximately 20 cm long pipe, stick or rod to indicate the direction towards the sun. The rod should be mounted in a distance of 6-8 cm from- and parallel to - the side of the outer box. The top end of the rod must be flat and cut perpendicular to the axis of the rod; neither must the top of the rod extent above the top of the oven. Most important, the axis of the rod must be perpendicular to the top of the pane. The shadow of the rod will guide the user to the right position of the oven. When the oven is positioned / adjusted for maximum power, the shadow from the rod will disappear - i.e. the shadow of the rod will merge with the rod itself.

If a washer (e.g. a used  $110 \text{ mm } \emptyset$  disk from a grinder) is placed at the bottom of the rod, the length of the shadow of the rod (at the washer) will, by the length of the shadow, show the degree of unalignment - and in which direction the oven must be shifted to become aligned.

For ease of the alignment, a dial (showing the angle of deviation from alignment) can be engraved (or painted) as circles on the washer.

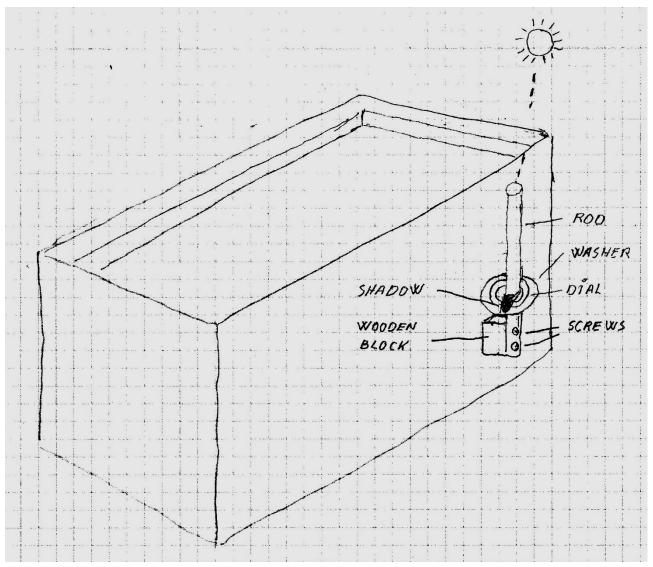


Fig. 9.

## 7.1 Tilting the oven to face the sun.

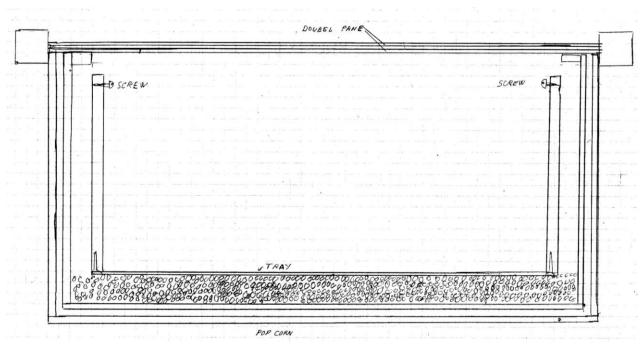


Fig. 10a

As stated earlier, the pane of the oven should be as near as possible to perpendicular to the sun. Hence, not only does the oven need to be able to rotate horizontally - it also has to allow to be tilted from upright position down toward horizontal position. No matter to which angel the oven needs to be tilted, there is a need for bottles, pots and pans to have a horizontal surface in the oven to stand on. Thus, a tray would be handy.

## 7.2 The horizontal alignment of the tray.

To be able to adjust a tray in the oven to stay horizontal, independent of how much the oven is tilted, I suggest to cover the bottom of the oven with a layer of popcorn or another granulate or filling material with a low thermal capacitance and which easily can be shifted around. I would therefore suggest to make a tray in a size, which fits into the inner box of the oven, when the oven is in upright position (i.e. not tilted).

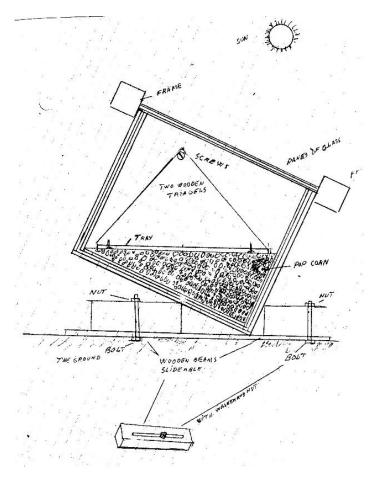


Fig. 10b

As for the length of the tray it must be able to pass by the two handles at the corners of the inner box.

The tray material could be: A 1-3 mm steel or aluminium sheet or >3 mm thick wood with a thin alufoil on top). To be able to lift the tray out of the oven, I suggest two triangularly shaped wooden boards (like bookends) mounted perpendicularly to the tray plate in each end of the tray (fig.10a). Place a round headed wood screw at the top of each of the two triangular boards (fig. 10b) at the end of the tray. Bend a steel rod or a thick steel wire into the shape of a hook to use for lifting out the tray by the two screws, when it is too hot to touch.

The least complicated compensation for tilting of the oven, will be for the oven to be pivoted around the edge of one of the long sides of the oven. To facilitate the adjustment of the oven relative to the sun, I suggest to use a bottom plate either of metal or from plywood. The oven is placed on the bottom plate, resting on the edge where the bottom of the oven meets the long side of the oven. Two beams - one on each side of the oven – are adjusted in their horizontal position to support and maintain the desired tilting angle of the oven. Each beam has one (or two) oblong hole(s) down along their middle. A bolt, a washer and a nut allows each of the beams to slide forth and back - or to be tighten to the bottom plate.

## 8. Thermal properties.

I cannot expect my readers to have a deeper insight in thermal properties. Hence, over the next couple of pages, I have included a small textbook on thermal properties. I hope to have eased the understanding by using more words and less mathematics to describe the issues. It is my hope, that this can be of some help to the readers. Sorry, if it is evident to you. If so - just skip it.

It is the difference in temperatures  $\Delta T$  between two points A and B, which causes heat to pass from: A to B (if the temperature of A is higher than the temperature of B) – and from B to A (if the temperature of B is higher than the temperature of A).

A flow of heat can be regarded as a flow of energy.

Power is measured in Watt [W] (equal to Joule per second). Power [P] is Energy (E) [Joule = J] per time unit (t) [measured in seconds = s] = [J/s]. Energy can also be measured in units of Calories. One Calorie = 4.184 Joule.

The thermal resistance ( $R_{th}$ ) [K/W or  ${}^{\circ}$ C/W] is defined as the temperature difference divided by the heat flow. As letters it can be expressed as  $\Delta T/\Delta P$ ; and it is measured in units of [ ${}^{\circ}$ C/W or K/W].

The thermal conductivity ( $\lambda_{th}$ ) is the opposite i.e. the inverse of the thermal resistance ( $R_{th}$ ). ( $\lambda_{th}$ ) = 1/( $R_{th}$ ).

The thermal conductivity ( $\lambda_{th}$ ) [W/°C or W/K] is defined as the heat flow divided by the temperature. As letters it can be expressed as  $\Delta P/\Delta T$ ; and it is measured in units of [W/K or W/°C].

The higher the resistance – the less heat passes. The higher the conductivity – the more heat passes.

Later I will describe how the thermal resistance is composed by the three thermal parameters:

A. Conductivity, B. Convection and C. Radiation.

- **8.1. The thermal capacitance** ( $C_{Th}$ ) is another thermal parameter. It describes a body's ability to store energy in the form of heat. Its unit is energy per degree Centigrade [J/ $^{\circ}$ C] or energy per Kelvin [J/K]. If we wish for the oven to heat up (- and to cool down-) quickly, the oven will have to have a low thermal capacitance.
- **8.2.** The transient temperature response and the thermal time constant. Assume that as a start the oven has the same temperature as the surroundings. Then, at the time t=0, it is placed in the sun. If there is no difference in the temperature  $\Delta T$  (between the inside of the oven and the surroundings), no heat will pass between the oven and the surroundings. Hence, all the power / heat from the sun will be converted into an increasing temperature- and stored in the thermal capacitance i.e. the oven and its contents. In other words: The energy is now stored in the thermal capacitance of the oven (including its contents of kitchen utensils, food and water etc.). This might be difficult to comprehend, hence, why it is explained in different ways.

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At the time t = 0, the rate of the rise in temperature per time unit (dT/dt) will be determined by the solar power P divided by the total thermal capacity  $C_{Th}$  of the oven and its contents. Thus, the more water and/or food (i.e. the higher a thermal capacitance) is placed in the oven, the longer will it take to raise its temperature.

As the temperature of the oven and its contents rises, the difference between the temperature inside the oven / and the temperature outside the oven increases. This will cause a bigger part of the heat from the sun to flow from the oven, (through the thermal resistance) to the surroundings. Therefore, less and less solar power will be left to heat the oven and its contents. Hence, – as can be seen at the figure below - the more the temperature of the oven with its contents (asymptotically) approaches its final temperature  $T_{Final}$  – the less power will be left over for "charging" the thermal capacitance. Thus, the temperature of the oven will rise slower and slower.

Ultimately (after eternally long time) - as the temperature in the solar oven has stabilized - the irradiated power from the sun to the oven, will be the same as the power leaving the oven to the surroundings - and the temperature will stop rising.

In other words - during the time it takes to establish this balance / equilibrium between the in-going and the out-going power, a still smaller part of the sun power going to the oven, will used for heating the oven and its contents (i.e. food, pots etc.). And - during the very same time, it takes for this balance / equilibrium to be established, an increasingly bigger part of the solar power, which enters the oven, will be "lost", i.e. it goes from the oven to the surroundings.

Physically - and mathematically - it can be expressed as: The final temperature of the oven will be equal to the ambient temperature ( $T_{Amb.}$ ) plus the solar power (P) multiplied by the thermal resistance ( $R_{th}$ ) between the interior of the oven and the ambient  $T_{Final} = T_{Amb.} + P \times R_{Th}$ .

Mathematically the temperature of the oven as a function of the time can be expressed as:

$$T(t) = T_{Amb.} + (P~x~R_{th})~(1-e^{-t/(Rth~Cth)}) = T_{Amb.} + (P~x~R_{th})~(1-e^{-t/\tau})$$
 - Where

T(t) is the temperature of the oven and its contents at the time t after the oven was exposed to the sun.  $T_{Amb.}$  is the ambient temperature (i.e. the temperature of the surroundings).

P is the solar power, which enters the oven.

 $R_{Th}$  is the thermal resistance between the interior of the oven and the ambient (i.e. the surroundings).  $C_{Th}$  is the thermal capacitance of interior of the oven and its contents.

 $\tau$  is the thermal time constant, and  $\tau = R_{Th} \times C_{Th}$ .

The shape of the temperature as a function of time T(t) (marked  $V_{CAP}$ ) is shown next page in fig. 11.

The horizontal axis represents the time t. The vertical axis represents the temperature T.

The crossing between the horizontal and the vertical axis is t = 0 and  $T = T_{Amb}$ .

The violet curve  $(V_{CAP})$  shows how the temperature of the oven asymptotically approaches

$$T_{Final} = T_{Amb.} + P R_{Th}$$
.

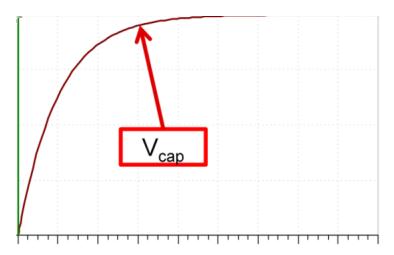


Fig. 11.

If we calculate  $(1 - e^{-t/(RTh \ CTh)})$  for  $t = 1 \cdot R_{Th} \cdot C_{Th}$ ,  $2 \cdot R_{Th} \cdot C_{Th}$ ,  $3 \cdot R_{Th} \cdot C_{Th}$  and  $4 \cdot R_{Th} \cdot C_{Th}$ , we will get:

$$T(K\tau) = T(t) = T_{Amb.} + (P \times R_{Th}) (1 - e^{-K})$$
, where  $K = 1, 2, 3$  and  $4 -$  which yields:

$$T(1\tau) = T_{Amb.} + 0.632 (P \times R_{Th}),$$

$$T(2\tau) = T_{Amb.} + 0.865 (P \times R_{Th}),$$

$$T(3\tau) = T_{Amb.} + 0.950 \; (P \; x \; R_{Th})$$
 - and

$$T(4\tau) = T_{Amb.} + 0.982 (P \times R_{Th}).$$

Similar relations apply to situations, where the influx of solar power to the oven is instantly cut off. In that case the temperature of the oven (as a function of time) can mathematically be written as:

$$T(t) = T_{Amb.} + (P \ x \ R_{th}) \ e^{-t/(Rth \ Cth)} \qquad \quad \text{- shown below - where}$$

T(t) is the temperature of the oven and its contents at the time t after the oven is no longer exposed.  $T_{Amb.}$  is the ambient temperature (i.e. the temperature of the surroundings).

P is the solar power, to which the oven was exposed just before the exposure stopped.

 $R_{Th}$  is the thermal resistance between the interior of the oven and the ambient (i.e. the surroundings).  $C_{Th}$  is the thermal capacitance of interior of the oven and its contents.

The shape of the temperature as a function of time T(t) (marked  $I_{CAP}$ ) is shown at the figure below. The horizontal axis represents the time t. The vertical axis represents the temperature T. The crossing between the horizontal and the vertical axes is t=0 and  $T=T_{Amb}$ . In this case  $T_{Amb}=0$ .

The violet curve shows, how the temperature of the oven asymptotically approaches  $T_{Final} = T_{Amb}$ .

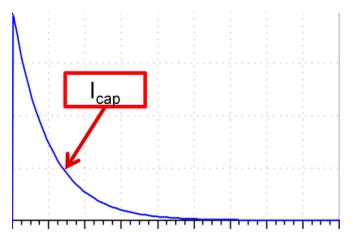


Fig. 12.

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**8.2. The thermal time constant.**  $(\tau_{Th})$  of the oven and its contents is the product of  $R_{Th}$  and  $C_{Th}$ . The thermal time constant  $\tau$  is measured in time units [seconds, minutes, hours or days].

The time constant is the time it takes for the oven to *increase* its temperature from

$$T_{Amb.}$$
 to  $T_{Amb.} + (T_{Final} - T_{Amb.}) (1 - e^{-1}) = T_{Amb.} + 0.632 (T_{Final} - T_{Amb.}).$ 

However, the time constant is also the time it takes for the oven to decrease its temperature

from 
$$T_{Start}$$
 to  $e^{-1}(T_{Start} - T_{Amb})$  =  $T_{Start} + 0.368(T_{Start} - T_{Amb})$ .

A thermal system consisting of a thermal resistance  $R_{Th}$  and a thermal capacitance  $C_{Th}$ , will have a time constant  $\tau$  equal to  $R_{Th} \cdot C_{Th}$ . For a rising step, the system will have a step response, which will have reached a temperature equal to 63.2% of its total change in temperature after *one*  $\tau_{Th}$ , - and 95% of its total change in temperature after *three*  $\tau_{Th}$ .

For a falling step the system will have a step response, which will have reached a temperature equal to 36.8% of its total change in temperature after *one*  $\tau_{Th}$ .

- and will have reached 5% of its total change in temperature after three  $\tau_{Th}$ .

## **8.3. The thermal resistance.** There exist three different kinds of heat transport:

- A. The conductivity,
- B. The convection and
- C. The radiation.

Each of these three kinds of heat transport must be considered if we want to calculate the thermal resistance between two points (A and B). All three kinds contribute to the total heat transport. Please, recall that the thermal resistance between point A and point B is the difference in temperature  $(T_B - T_A)$  between the two points (A and B), divided by the total amount of heat  $(\Sigma P)$  flowing between the two points.

Thus, to calculate the thermal resistance between point A and point B, I must first decide by which temperatures  $T_A$  and  $T_B$  at point A and point B, I want to do the calculation.

Once  $T_A$  and  $T_B$  have been determined, I must – for each of the three contributions; i.e. for the *conduction*, for the *convection* and for the *radiation* - calculate how much heat (= power) must flow between A and B to generate the desired difference between the temperatures  $T_A$  and  $T_B$ .

Next step is to add up the three heat flows – the one from the *conduction*, the one from the *convection* and the one from the *radiation*. Knowing the sum of each of these three heat flow contributions (denoted  $\Sigma P$ ) and knowing the difference in temperature between the desired temperatures  $T_A$  and  $T_B$ , I can now calculate the resulting thermal resistance  $R_{Th} = |T_B - T_A|/\Sigma P$ .

Please notice, that both the convection and the radiation are non-linear functions. Never the less, out of convenience (and due to the generally low accuracy in thermal designs) - in many calculations we pretend these non-linear functions to behave as linear functions – and use them as such.

#### 8.3.1. The thermal conduction.

The thermal conduction is the inverse of the thermal resistance. It is denoted k and it is measured in units of [W/K]. Most people will have experiences with thermal conduction. E.g. when holding on to one end (A) of a metal spoon, where the other end (B) is in the soup. Soon, the heat will have spread through the spoon, and the (A) end of the spoon will become too hot to hold on to.

Heat transport through a spoon can be visualized by comparing it to water passing through a pipe:

- 1. The bigger the cross-section area of the pipe the lower the resistance to the water flow will be.
- 2. Likewise the bigger the cross-section area of the handle of the spoon the lower the thermal resistance to the heat flow will be.
- 3. And the longer the pipe the higher the resistance of the flow of the water will be. Likewise - the longer the spoon handle - the higher the thermal resistance to the heat flow will be.

There are two more parameters / material constants, which determine the *conductive* part of the thermal resistance. The first is *the specific thermal resistance* ( $R_{\lambda}$ ) [Km/W].  $R_{\lambda}$  is (almost) a constant, which depends on the material of the object. To find *the thermal resistance* ( $R_{Th}$ ) [K/W] of an object, you must multiply  $R_{\lambda}$  (for that object with its length and divided it by its cross section area (i.e. the cross section area of the object).

 $R_{\lambda}$  is generally only *almost* constant. Generally, it changes with the temperature. The parameter, which describes this dependence, is called *the temperature coefficient* of  $R_{\lambda}$  for a given specific material. The temperature coefficient is generally called  $(\alpha_{R\lambda})$  and its unit is  $[K^{-1} \text{ or } {}^{\circ}C^{-1}]$ . Thus,

 $R_{\lambda}(T) = R_{\lambda Ref} (1 + \alpha_{R\lambda} (T - T_{Ref}))$ 

And finally we get:

 $R_{Th}(T) = R_{\lambda Ref} (1 + \alpha_{R\lambda} (T - T_{Ref})) L / A$ 

where:

 $R_{Th}(T)$  is the thermal resistance of an object as function of its the temperature T.

L is the length of the object.

A is the cross section area of the object.

 $R_{\lambda}(T)$  is the specific thermal resistance of the material at the temperature T.

 $R_{\lambda Ref}$  is the specific thermal resistance of the material at the reference temperature  $T_{Ref}$ 

- which is usually 20°C - but occasionally 0°C.

 $\alpha_{R\lambda}$  is the temperature coefficient of the specific thermal resistance (at a specific temp.).

T is the actual temperature.

 $T_{Ref}$  is the reference temperature, by which  $\alpha_{R\lambda}$  is defined, found or given.

Mind!  $R_{\lambda Ref and} \alpha_{R\lambda}$  applies to heat transported as *conduction* - not to *convection* and *radiation*.

From here we find the thermal conduction (as function of the temperature) as the inverse of the thermal resistance (as function of the temperature); i.e. the thermal conduction  $k(T) = 1 / R_{Th}(T)$ .

**8.3.2. The convection.** The second kind of heat transport is convection. Convection occurs, when air passes over, along- or below a body, which has a temperature that differs from the temperature of the surrounding air. When the air is either heated or cooled by the body, the density of the air will change (relative to the surrounding air). If air has been heated, buoyance will cause it to raise. Opposite, if air has been cooled down, it will sink. In either case new air with another temperature will be drawn towards the body and it will either be cooled off - or heated.

The heat (i.e. the power P<sub>Convection</sub>; measured in [W]), which is transported by convection from- or tothe surface of a body, can be approximated by the equation:

$$P_{\text{Convection}} = K * A * \Delta T^{1.25} / L^{0.25} \implies \Delta T = (P_{\text{Convection}} * L^{0.25} / (K * A))^{0.8}$$

### - where:

 $\Delta T$  is the difference in temperature between the body and the ambient air in units of [K or  ${}^{\circ}C$ ].

## For the top side:

L [m] = is the square root of the product of the largest- and the smallest- measurements. A is the area of the top side  $[in m^2]$ .

### For the vertical sides:

L [m] = is /are the height(s) of the vertical sides. A is the sum of the areas of the vertical sides [ $m^2$ ].

## For the underside:

L [m] = is the square root of the product of the largest- and the smallest- measurements. A is the underside area in  $[m^2]$ .

K is a constant. For the top side it is 1.78; for the vertical sides is 1.37 and for the underside 0.96.

**8.3.3. The radiation.** The third and last kind of heat transport is the radiation. The lowest temperature in existence is the absolute zero, where all molecular movements come to a halt and all radiation stops. Radiation occurs when a body has a radiation coefficient > 0.0 and its temperature exceeds the absolute zero. The radiation increases by the absolute temperature (measured in Kelvin) [K] to the power of four. The absolute zero is  $-273.15^{\circ}\text{C} = 0.0 \text{ K}$ . ( $0^{\circ}\text{C} = 273.15 \text{ K}$ ). Thus, one degree, counted in units of Celsius [ $^{\circ}\text{C}$ ], has the same magnitude as one degree counted in unities of Kelvin [K].

Two bodies A and B – both with temperatures higher than  $-273.15^{\circ}$ C - which can "see" each other - will mutually exchange heat radiation. This implies that a body A and a body B both will radiate heat (to their surroundings and to each other). The resulting heat radiation is the difference between the two radiations.

**The radiation coefficient.** The total heat radiation from a body is proportional to its surface area, multiplied by the radiation coefficient  $C_s$  for the surface of the body, which radiates the heat. The radiation coefficient  $C_s$  for an absolute black body is  $5.67 * 10^{-8}$  W m<sup>-2</sup> K<sup>-4</sup>.

The radiated heat / power (P) [W] from a body is:

$$P = C_R * C_s * A * T^4 \implies T = (P / (C_R * C_s * A))^{-4}$$

where:

 $C_s$  is the radiation coefficient (= 5.67 \*  $10^{-8}$  W m<sup>-2</sup> K<sup>-4</sup> for an absolute black body)

A is the area  $[m^2]$  of the surface of the body, which radiates the heat.

T is the absolute temperature of the body in Kelvin [K] (i.e. the temperature in  ${}^{\circ}\text{C} + 273.15 \text{ K}$ ).

 $T_{Amb.}$  is the absolute temperature of the ambient in Kelvin [K] (i.e. the temperature in  ${}^{\circ}C$  + 273.15 K).  $C_R$  is a constant (between 0.0 and 1.0) depending on the surface of the body / material.

 $C_R$  describes the reflection from a surface of a body, i.e. the ability of that surface to absorb and radiate heat. It is 1.0 for an absolute black body. If you look at fig.7, you will see an almost black surface. Hence, why I have estimated a  $C_R = 1,0$  for the glass area of the lid.

For a blank (fully reflecting) surface / body (like a mirror) C<sub>R</sub> is 0.

For white bodies C<sub>R</sub> nearly (almost) 0.

If the ambient temperature is  $T_{Amb.}$ , the net heat radiation  $P_{Net}$  from the body will be the difference between the radiated and the irradiated heat / power from the body - and from the ambient.

i.e. 
$$P_{Net} = C_R * C_s * A * (T^4 - T_{Amb.}^4)$$
.

## 8.4. Configuration and calculations of the thermal equivalent of the solar oven.

**8.4.1. Kirchhoff's law and power accounts.** Kirchhoff's law states, that the sum of electrical charges, which goes into a junction - is identical to the sum of charges leaving the same junction. What goes in - must come out! This also applies to a thermal system, such as the solar oven.

If we want an infinite exact calculation, we have to divide the oven into infinite many points, regard each point as a junction, and then calculate all the heat flows between each of the infinite many junctions to all the rest of the infinite other junctions. It is evident, that one must cut corners and simplify the model of the oven to a manageable number of junctions. I have chosen a model with four junctions as a suitable trade off.

One of the junctions in a thermal model will generally be chosen as the reference junction. In this model the ambient temperature (or you could say the temperature of the surroundings or the environment) is chosen as the natural reference junction.

In electrical systems the variables linked to the junctions are the voltages of- and the currents to- and from the junctions. Likewise, in a thermal system, the variables linked to the junctions are the temperature of- and the heat flows to- and from the junctions.

# - Similarly:

In a thermal system, a thermal resistor compares to an electric resistor in an electric system.

In a thermal system, a thermal capacitance compares to an electric capacitance in an electric system.

In a thermal system, a heat flow compares to an electric current in an electric system.

In a thermal system, a temperature compares to a voltage in an electric system.

-----

In this analysis, the thermal configuration of the solar oven consists of four junctions:

- A. The interior of the solar oven including its contents of pots, food, water etc.
- B. The outside of the glass.
- C. The outside surface of rest of the solar oven.
- D. The ambient i.e. the surroundings of the solar oven. This is the reference junction.

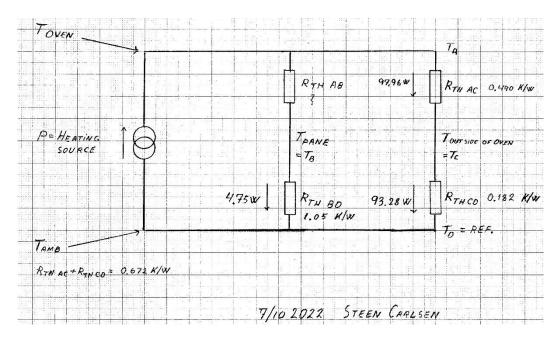
The configuration of the thermal components between the four junctions:

- 1. The heat flow from the sun into the oven. Junctions A & D.
- 2. The thermal resistance between the surface of the pane and the ambient. Junctions B & D.
- 3. The thermal resistance between the surface of the rest of the oven and the ambient. C & D.
- 4. The thermal resistance between the inside of the oven and the pane. Junctions A & B.
- 5. The thermal resistance between the inside and the outside of the oven minus the pane. A & C.
- 6. The thermal capacitance between the inside of the oven and the ambient. Junctions A & D.

## 8.4.2. The diagram of the thermal model of the solar oven.

In the diagram of the model (fig. 13. shown below), only the components relevant for steady state operation (i.e. when the oven is in thermal equilibrium) are shown. Hence, why the thermal capacitance (6) between the interior of the oven and the ambient has been omitted. If you want to include the thermal capacitance, it shall be drawn as a capacitor mounted across (i.e. parallel with) P. P is the heating source.

Once the system has reached  $T_{Final}$  (= its thermal equilibrium) the thermal capacitance will no longer have any impact. The thermal capacitance will be addressed later.



The diagram of the electrical equivalent to the thermal system of the solar oven. Fig. 13.

Looking at the diagram, one sees that, if we aim for a high internal temperature in the oven, we should wish for:

- A. A high temperature  $T_{Amb.}$  of the surroundings (i.e. the ambient) plus
- B. A high amount of power P (= heat) going from the sun into the oven and
- C. A high thermal resistance from the inside of the oven out to the surroundings (i.e. ambient).

**8.5.3.** The power (i.e. heat) irradiated into the oven. If you are designing - or making improvements - to a solar oven, you will surely want to know the temperatures in the junctions of the diagram of the oven. In that case, here are some hints.

A solar oven is designed for the sun to be its power source (just to state the obvious!). But the irradiated heat from the sun generally fluctuates so much, that before a measurement can be taken its value has changed. Thus, during my tests, I have switched between two different heating sources – partly the sun and partly an electric cooking plate. When you use electrical heating, do cover the glass, to prevent erroneous test results due to irradiation of unaccounted light.

Hence, a part of the tests and the measurements have been done in my sitting room at a fixed ambient temperature and using the electric power from the mains, a vario transformer and a cooking plate as controllable heating source. Later, I have realized, that it would had been better if I had used an UPS (UPS = Uninterruptable Power Supply) to avoid the fluctuations in the mains voltage. Using an UPS would have improved the stability / control of the power dissipation in the cooking plate.

I know, that not many people have access to a cooking plate, a vario transformer let alone an UPS. Even without an UPS you can still get some fair results using a number of low power (5 / 10 / 15 / 25 W) electrical appliances. It is an advantage to use many heating sources to distribute the heat more evenly inside the oven. One of the glasses in my pane cracked due to too an unevenly distributed heat from my cooking plate. The cooking plate, which I used to imitate the sun supplied 0-250W of heat.

Electrical lamps are a good choice. Most bulbs have sockets, which allow for swift and easy connection and disconnection. A specific advantage of using incandescent lamps/bulbs is, that their inherent temperature coefficient - to some extent - compensates for the impact, which the variations in their supply voltage will have on their power dissipation. For an improved distribution of the heat use a fan (e.g. from a scrapped computer). Please, make sure to use the voltage, for which the fan is marked (often it is 12V). Do not forget to measure and add the electrical power consumption of the fan(s) to the power dissipated by the lamps.

If you are in a place with no access to mains power, you might use the 12V / 14V from a car battery and low power incandescent bulbs from the stop-light, indicator-lights and number plate lights. Most traditional types of bulbs for these lights will have their voltage and their power stamped / engraved in their metallic sockets. If you want to draw 240W at 12V, the current will reach 20A. By 20 A you will require a wire gauge  $\geq 4$  mm<sup>2</sup>. For a current I = 10 A the wire gauge  $\geq 0.75$  mm<sup>2</sup>.

Make sure that the wire gauges are sufficient to cope with the currents (I[A] = P[W] / V[V])..

For a cloudless sky, the irradiated power from the sun into the oven is:  $P_{Sun} = I_{Sun} \times A \cos \alpha$ ;

where P<sub>Sun</sub> is the irradiated heat from the sun.

Isun is the intensity of the sun at the actual site and time-,

A is the area of pane / opening into the inner box of the oven

 $\alpha$  is the angle between a line from the oven to the sun and a line perpendicular to the pane.

For the present solar oven:  $P_{Sun} = I_{Sun} \times A \cos \alpha = 1000 \text{ W/m}^2 \times (0.30 \text{ m} \times 0.49 \text{ m}) \times \cos 0^\circ = 147 \text{ W}.$ 

- Of which an unknown fraction of the power outside the visual spectrum might be lost (filtered away) by the glass in the oven.

## 8.5. Calculating the thermal resistances using the temperatures and the heat flow contributions.

In the following, I have calculated / estimated the thermal resistances in the solar oven, based on: A. The thermal model described in section 8.4.2.

B. The thermal and mechanical measurements I have made in my sitting room and in my allotment.

The thermal resistances for the present prototype of the solar oven have been calculated as described in section 8.3. "The thermal resistance". In section 8.3. we found the thermal resistance between point A and point B by dividing the difference in their temperatures by the sum of the heat flows caused by the conduction, the convection and the radiation.

This is also, the procedure, which has been applied by each of the calculations of the thermal resistances of the oven – presented on the next page. At one point, the calculations on the next page deviate from the expected. I have – I guess wrongly – assumed the touching surfaces (between the inner and the outer box of the solar oven) to be so small, that their thermal resistances would be so high that it would justify not to count them in (in the sum of the other thermal conductivities).

When looking through the glass in the pane of the solar oven – see fig. 5 - it is difficult to see anything but darkness. This, I interpreted as a near to complete absorption of all incident light. Thus, in the calculations of the radiated losses I have estimated the radiation coefficient  $C_R$  of the glass in the pane to be 1.0.

Apart from the top side of the oven - most outside surfaces are white. Hence, I have (presumably conservatively) guessed their radiation coefficient to be 0.5. I believe the real figure to be nearer to 0.2.

I have gone through the recorded temperatures (in the appendix) and found that, the most suitable data set (i.e. the one with least uncertainty) is the data set recorded at 9.26 a.m. on August 11th. 2022.

The recorded power and temperatures of the data set, which was used, were:

P = 125.1 W, (supplied by the cooking plate)

 $T_{Amb.} = 25^{\circ}C.$ 

 $T_{Pane} = 30^{\circ}C$ .

 $T_{Ext. Box} = 42^{\circ}C.$ 

 $T_{Oven} = 90^{\circ}C.$ 

This page contains the calculations and the recapitulation of the components in the diagram of the thermal model of the oven – as shown in section 8.4.2. (The equations for the convection and the radiation can be found in the sections "8.3.2 The convection." and "8.3.3 The radiation.")

The indexes A, B, C and D referee to the junctions in the diagram.

1. 
$$P_{Sun AD}$$
 = 125.1 W.

2. Thermal resistance from the pane to ambient -  $R_{Th BD}$ :

$$P_{\text{Convection BD}} = 1.78 \text{ x } (0.575 \text{ m x } 0.38 \text{ m})$$

$$(30^{\circ}C - 25^{\circ}C)^{1.25}/(0.575 \text{ m} \times 0.38 \text{ m})^{0.5})^{0.25} = 2.49 \text{ W}.$$

$$P_{Radiation\;BD}\!=0.5\;x\;5.67\;*\;10^{\text{-8}}\;W\;m^{\text{-2}}\;K^{\text{-4}}\;x\;(0.49\;m\;x\;0.30\;m)$$

$$x ((30^{\circ}C + 273^{\circ}C)^{4} - (25^{\circ}C + 273^{\circ}C)^{4})$$

= 2.26 W.

 $P_{Conduction \; BD}$  is assumed neglectable.

0.0 W.

$$R_{Th BD}$$
: =  $\Delta T / \Sigma P = T_{Pane} / (P_{Convection BD} + P_{Radiation BD})$ 

= 
$$(30^{\circ}\text{C} - 25^{\circ}\text{C}) / (2.49 \text{ W} + 2.26 \text{ W}) = 5 \text{ K} / 4.75 \text{ W} = 1.05 \text{ K/W}.$$

3. Thermal resistance from the outer vertical- and bottom-sides of the solar oven to ambient - R<sub>Th CD</sub>:

$$P_{\text{Convectio CD Vertical}} = 1.37 \text{ x} ((2 \text{ x } 0.575 \text{ m} + 2 \text{ x } 0.38 \text{ m}) \text{ x } 0.32 \text{ m})$$

$$x (42^{0}C - 25^{0}C)^{1.25} / (0.32 \text{ m}^{0.25}) = 38.43 \text{ W}.$$

 $P_{\text{Convectio CD Bottom}} = 0.96 \text{ x } (0.575 \text{ m x } 0.38 \text{ m})$ 

$$x (42^{0}C - 25^{0}C)^{1.25} / (0.575 \text{ m x } 0.38 \text{ m})^{0.5})^{0.25} = 8.76 \text{ W}.$$

$$P_{Radiation\;CD}\!=0.5\;x\;5.67\;*\;10^{\text{-}8}\;W\;m^{\text{-}2}\;K^{\text{-}4}$$

$$x (0.575 \text{ m} \times 0.38 \text{ m}) + ((2 \times 0.575 \text{ m} + 2 \times 0.38 \text{ m}) \times 0.32 \text{ m})$$

$$x ((42^{0}C + 273^{0}C)^{4} - (25^{0}C + 273^{0}C)^{4}) = 46.09 W.$$

 $P_{\text{Conduction }\text{CD}}$  is assumed neglectable.

0.0 W.

$$R_{Th\;CD}\text{:}=\Delta T\,/\,\Sigma P=T_{Pane}\,/\,(P_{Convection\;BD}+\,P_{Radiation\;BD})$$

= 
$$(42^{0}\text{C} - 25^{0}\text{C}) / (38.43\text{W} + 8.76\text{ W} + 46.09\text{ W}) = 17\text{ K} / 93.28\text{ W} = 0.182\text{ K/W}.$$

4. Thermal resistance from the interior of the solar oven to the outer surfaces of the solar oven -  $R_{Th AC}$ :

$$P_{\text{Convectio AC Vertical}} = 1.37 \text{ x } ((2 \text{ x } 0.515 \text{ m} + 2 \text{ x } 0.325 \text{ m}) \text{ x } 0.233 \text{ m})$$

$$x (90^{\circ}C - 42^{\circ}C)^{1.25} / (0.233 \text{ m}^{0.25}) = 48.76\text{W}.$$

 $P_{Convectio \ AC \ Bottom} = 0.96 \ x \ (0.515 \ m \ x \ 0.325 \ m)$ 

$$\times (90^{\circ}\text{C} - 42^{\circ}\text{C})^{1.25} / (0.515 \text{ m} \times 0.325 \text{ m})^{0.5})^{0.25} = 25.38 \text{ W}.$$

$$P_{Radiation AC} = 0.1 \text{ x } 5.67 * 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

$$x (0.515 \text{ m} \times 0.325 \text{ m}) + ((2 \times 0.515 \text{ m} + 2 \times 0.325 \text{ m}) \times 0.233 \text{ m})$$

$$x ((90^{\circ}C + 273^{\circ}C)^{4} - (42^{\circ}C + 273^{\circ}C)^{4}) = 23.81 \text{ W}.$$

P<sub>Conduction AC</sub> is neglectable

 $0.0 \mathrm{W}$ .

$$R_{Th~AC} := \Delta T \, / \, \Sigma P = T_{Core} \, / \, \left( P_{Convection~BC} + P_{Radiation~BC} \right)$$

$$= (90^{\circ}\text{C} - 42^{\circ}\text{C}) / (48.76 \text{ W} + 25.38 \text{ W} + 23.81 \text{ W}) = 48 \text{ K} / 97.96 \text{ W} = 0.490 \text{ K/W}.$$

## 8.5.1. Check of balance in the account of: the heat flows, thermal resistances and temperatures.

Recalling Kirchhoff's law, we realize, that if we assume to have a thermal equilibrium, the power entering the oven from the sun will be equal to the power dissipated from the oven – by convection, by radiation and by conduction.

I have checked the measurements and calculations by comparing the thermal resistances, which I calculated in the last section, to the temperatures, which were recorded at 9.26 a.m. on August 11th. 2022 (and which can be found in the appendix). This data set also provides the temperatures, on the base of which, the thermal resistances are calculated. The values of the calculated thermal resistances can also be found at the diagram of the thermal system of the solar oven (fig. 13, section 8.4.2.). A graph of the thermal resistances as a function of the temperature is found in fig. 15 section 9.3.

The temperatures of the four junctions in the model were:

 $T_A$  is the interior temperature of the oven = 90°C.

 $T_B$  is the temperature of the outer surface of the pane = 30°C.

 $T_C$  is the temperature of the outer surface of the oven =  $42^{\circ}$ C.

 $T_D = T_{Amb.}$  is the ambient temperature = 25°C. (Junction D =  $T_{Amb.}$  is the reference in this model.)

The four calculated thermal resistances were:

 $R_{Th AB} = so far unknown.$ 

 $R_{Th BD} = 5 \text{ K} / 4.75 \text{ W} = 1.05 \text{ K/W}.$ 

 $R_{Th AC} = 48 \text{ K} / 97.96 \text{ W} = 0.490 \text{ K/W}.$ 

 $R_{Th CD} = 17 \text{ K} / 93.28 \text{ W} = 0.182 \text{ K/W}.$ 

Looking at the diagram, one can see that the power, which flows through  $R_{Th\ AC}$  should and must be identical to the power, which flows through  $R_{Th\ CD}$ .

Likewise, the power, which flows through  $R_{Th AB}$  should and must be identical to the power, which flows through  $R_{Th BD}$ .

And – recalling Kirchhoff's law – the sum of the power, which flows through  $R_{Th AB}$  and  $R_{Th BD}$ , plus the power, which flows through  $R_{Th AC}$  and  $R_{Th CD}$ , should and must be identical to the power, which flows through / comes from the heating source (P) (i.e. the left vertical branch in the diagram.)

I start by finding the missing  $R_{Th AB}$ . Knowing that the power flowing through  $R_{Th AB}$  should and must be identical to the power flowing through  $R_{Th BD} = 4.75$  W, and that the temperature difference across  $R_{Th AB}$  is (90°C - 30°C =) 60 K, we find that  $R_{Th AB} = 60$  K / 4.75 W = 12.63 K/W.

Having just used the power, which flows through  $R_{Th \, BD}$  to calculate  $R_{Th \, AB}$  means there is no sense in comparing the power flowing through the two thermal resistances  $R_{Th \, AB}$  and  $R_{Th \, BD}$  – we have already defined it to 4.75 W.

The power, which flows through  $R_{Th\ AC}$  was calculated to 97.96 W. The power, which flows through  $R_{Th\ CD}$  was calculated to 93.28 W. The deviation is 5 %, which - for the accuracy to be expected for these measurements - is surprisingly good.

Finally, I must compare the sum of the power, which flows through  $R_{Th AB}$  and  $R_{Th BD}$ , plus the power, which flows through  $R_{Th AC}$  and  $R_{Th CD}$ , to the power, which flows through / comes from the heating source (P).

The power, which flows through R<sub>Th AB</sub> and R<sub>Th BD</sub> I have defined to be 4.75 W.

The power, which flows through R<sub>Th AC</sub> and R<sub>Th CD</sub> was found to 97.96 W and 93.28 W, respectively.

Depending on whether we choose 97.96 W or 93.28 W - the total sum of the power flowing from the heating source, through the oven and on through the two right branches in the diagram and out to the ambient ( $T_D$ ), will be 4.75 W + 93.28 W = 98.03 W - or 4.75 W + 97.96 W = 102.71 W.

The power source, which was a cooking plate - dissipated 125.1 W. I assume the accuracy of this power to be in the order of +/- 2-3% - or better. As for the accuracy of the power leaving the oven (i.e. the two values 98.03 W and 102.71 W), I estimate its accuracy to be +/-10% - or better. Even in the best case the two figures are more than 16% out of balance. I might have been wrong, when I assumed the conductive parts of the heat flows to be negligible in the calculation of the thermal resistances.

This difference, in the figures for the in- and the outgoing power of the oven, to me indicates, that I was wrong, when I assumed the conductive parts of the heat flows to be negligible. I now think, that the conductive part of the heat flow, contributes to the calculated losses with an extra power / heat of approximate 25 W (with an estimated tolerance of this power in the order of  $\pm$ 10%).

Given the sizes of the cross section areas of the threading points between the inner- and outer box - and between the outer box and the surroundings – I am, never the less, still astonished about the magnitude of the conductive losses.

If we calculate the thermal resistance R<sub>Th AD</sub> based on:

P - i.e. the known electrical (and, thus, also the thermal-) power dissipated in the oven – and

 $\Delta T$  – i.e. the difference in the temperature inside the oven and the ambient temperature –

We get 
$$R_{Th AD} = \Delta T / P = (90.1^{\circ}C - 25^{\circ}C) / 125.1 W$$
 = 0.52 K/W.

The resulting resistance R<sub>Resulting</sub> of two thermal resistances R<sub>M</sub> and R<sub>N</sub> in series is

$$R_{Resulting} = R_M + R_{N.} - and$$

The resulting resistance R<sub>Resulting</sub> of two thermal resistances R<sub>O</sub> and R<sub>P</sub> in parallel is

$$R_{\text{Resulting}} = (R_{\text{O}}^{-1} + R_{\text{P}}^{-1})^{-1}.$$

If in instead, we calculate, 
$$R_{Th AD}$$
 as =  $((R_{Th AB} + R_{Th BD})^{-1} + (R_{Th AC} + R_{Th CD})^{-1})^{-1}$  — we get:  
=  $((12.63 \text{ K/W} + 1.05 \text{ K/W})^{-1} + (0.490 \text{ K/W} + 0.182 \text{ K/W})^{-1})^{-1}$  =  $0.64 \text{ K/W}$ .

I believe, that this difference can partly be ascribed to the unaccounted conductive part of the resistance of the latter (the 0.64 K/W) – and partly due to the variations (i.e. un-linearity) of the thermal resistances as a function of the temperature. The un-linearity will be addressed in the next section (9.), and can be found at the plot in fig. 15 in section 9.3.

## 9. Issues of measuring the thermal properties of a solar oven.

To me – and I guess to anyone who wants to design, develop, improve or build a solar oven - it is important to be able to quantify the merits of the solar oven. For solar ovens the most important merits / properties to be identified are:

- A. The maximum difference in temperature (A) between the interior and the surroundings of the oven, which the solar oven is able to achieve. This difference I call A (=  $P_{Sun} \times R_{th}$ ) and
- B. The time it takes for the oven to achieve a given fraction of that difference (A) in temperature. This time is quantified in the oven's time constant called  $\tau$  (=  $R_{th}$  x  $C_{th}$ ).

For a solar oven to heat up to a sufficient temperature to find A and  $\tau$  - in theory takes infinitely long – and in praxis unbearable long. Hence, it would be a major advantage to find an analytical mathematical expression for A and  $\tau$ , based on measurements of the temperature (T) of the oven as a function of the time (t).

I managed to find two exponential equations with A and  $\tau$  expressed in terms of  $(t_1, T_1)$  and  $(t_2, T_2)$ . Not knowing if a general analytical solution to the exponential equations exists, I tried to solve the equations for A and  $\tau$  - and for *any* ratio between  $t_1$  and  $t_2$  (- except for the ratios of: 0.0, 1.0 and  $\infty$ ).

When trying to isolate and to express A only in terms of  $(t_1, T_1)$  and  $(t_2, T_2)$ , I stalled. I realized that people with more flair for mathematics than me were required for the task. I then addressed some friends for help to solve the equations. Two old friends – Torkil and Gunnar both in their early nineties and both with university degrees in mathematics - found a solution on the condition that  $t_2 = 2 t_1$ . Their solution is presented on the following pages. After Torkil and Gunnar's solution, you will find a presentation of my wrecked attempt to find a *general* solution.

Meanwhile – in order to move on – and without knowing if any of my friends would be able to find and return with an analytic solution - I turned to a less accurate approach. Knowing the shape of an exponential curve I compared that to the graph / plot of my measurements of the temperature of the oven as a function of time. Based on these comparisons I - by eye - estimated the final / maximum temperature  $T_{Final}$  of the oven ; and based on that I found A and the associated thermal time constant  $\tau$ . The values, which I in this unprofessional way obtained by eye – never the less seemed plausible.

Assuming that someone can- or wish to use the work done by Torkil and Gunnar – and to less extent my calculations – as a lever or a stepping stone for a general analytic solution, I have translated and typed Torkil's and Gunnar's handwritten solution from Danish into English. I am impressed and most grateful for their achievements, which are presented at the next pages - followed by my own attempt.

Finally – after the pages with my own wrecked attempt to find an analytic solution to P x R<sub>th</sub> and  $\tau$  out of  $(t_1, T_1)$  and  $(t_2, T_2)$  - you will find the results of my "by eye" estimations.

# 9. On the properties of solar ovens. - By Torkil and Gunnar.

The quality of a solar oven is determined by two properties: How warm will it get, when placed in the sun, and how long will it take to heat up. If we measure the time from the moment, when the oven is exposed to the sun, and the temperature of the oven relative to the ambient temperature, the temperature of the oven as a function of the time can be described by a graph as the one shown at Figure 1. Here x could be the time and y the temperature.

If we define:

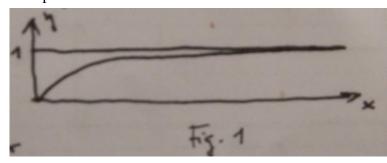
(1) 
$$y = 1 - e^{-x}$$
 for  $x \ge 0$ ,

we will get a graph, where

(2) 
$$y = 0$$
 for  $x = 0$ 

(3) 
$$y \to 1$$
 for  $x \to \infty$ .

But this is not flexible enough. The



horizontal asymptote should not be y = 1, but Y = A, where A is the temperature, which the oven approaches asymptotically. This is achieved by setting

(4) 
$$y = T/A$$
 - yielding

(5) 
$$T = A (1 - e^{-x})$$
 for  $x \ge 0$ ,

where now it is T, which is the temperature.

But this is still not flexible enough, as the speed, by which the temperature rises, is

(6) 
$$dT/dx = A (0 - (e^{-x})) = A e^{-x}$$
.

Hence, for any oven with a given A, the temperature will rise with the same slope. We also note that

(7) 
$$dT/dx = A$$
 for  $x = 0$ .

Thus, at origo the slope of the graph will always be A.

We now define

(8) 
$$x = t / \tau$$
,

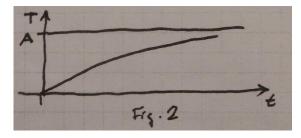
where  $\tau$  is a positive constant; which gives us

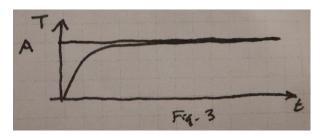
(9) 
$$T = A (1 - e^{-t/\tau}), t > 0.$$

Finding the slope of the graph by differentiation we get

(10) 
$$dT/dt = dT/dx \cdot dx/dt = A e^{-x} \cdot 1 / \tau = A \cdot e^{-t/\tau} \cdot 1 / \tau$$
.

Thus, the graph is flat for large values of  $\tau$ , and it is steep for small  $\tau$  (- but always positive).





To evaluate a given solar oven, we wish to find A and  $\tau$ , by placing the oven in the sun and observe T for various values of t. One single observation yields  $T = T_1$  for  $t = t_1$ . Hence,

(11) 
$$T_1 = A (1 - e^{-t1/\tau}).$$

This single equation is clearly not enough to find both A and  $\tau$ .

A second observation yields  $T = T_2$  for  $t = t_2$ , thus

(12) 
$$T_2 = A (1 - e^{-t2/\tau}),$$

And we might be able to determine A and  $\tau$  based on the two equations (11) and (12).

But it will not be simple, as A appears as a factor outside a bracket, while  $\tau$  is hidden as the index in an exponential factor within the very same bracket. It will probably be a good idea to rearrange (9) a bit more before we insert  $t_1$ ,  $T_1$ ,  $t_2$  and  $T_2$ . From (9) we find

- (13)  $A T = A e^{-t/\tau}$  hence,
- (14)  $(A T)/A = e^{-t/\tau}$
- (15)  $1 T/A = e^{-t/\tau}$
- (16)  $\ln (1 T/A) = -t/\tau$

(We note that, as T < A so  $T/A < 1 \Rightarrow 1 - T/A > 0$ . Thus the term  $\ln (1 - T/A)$  does make sense.)

Finally, we rewrite (16) to

(17) 
$$t = -\tau \ln (1 - T/A)$$

Here we have an equation (17), into which we can insert  $t_1$ ,  $T_1$ ,  $t_2$  and  $T_2$ .

Inserting  $t_1$ ,  $T_1$ ,  $t_2$  and  $T_2$ . we obtain

(18) 
$$t_1 = -\tau \ln (1 - T_1/A)$$

(19) 
$$t_2 = -\tau \ln (1 - T_2/A)$$

Which seem much more manageable than (11) and (12). By division, we obtain

(20) 
$$t_2 / t_1 = \ln (1 - T_2 / A) / \ln (1 - T_1 / A)$$

If we could get rid of  $t_1$  and  $t_2$  we would be left with an equation with A,  $T_1$  and  $T_2$ , in which we might be able to find A expressed by  $T_1$  and  $T_2$ , which is what we wish for. And it is indeed possible, as we can choose the points in time  $t_1$  and  $t_2$  when we measure the temperatures  $T_1$  and  $T_2$ . If we choose  $t_1$  and  $t_2$  so that  $t_2 = 2$   $t_1$ , then

(21) 
$$t_2 / t_1 = 2$$

which, together with (20) leads to

(22) 
$$2 = \ln (1 - T_2/A) / \ln (1 - T_1/A)$$
.

Hence,

(23) 
$$\ln (1 - T_2/A) = 2 \ln (1 - T_1/A) = \ln ((1 - T_1/A)^2) = \ln (1 - 2T_1/A + T_1^2/A^2)$$

which leads to

(24) 
$$1 - T_2/A = 1 - 2T_1/A + T_1^2/A^2 = >$$

(25) 
$$-T_2/A = -T_1A + T_1^2 = >$$

(26) 
$$(2T_1 - T_2) A = T_1^2 = >$$

(27) 
$$A = T_1^2 / (2T_1 - T_2) A =$$

(Please observe, that the graph of equation (9) turns clockwise for increasing t. Hence,  $2T_1 - T_2 > 0$ .)

Based on (27) we have found A expressed by  $T_1$  and  $T_2$ . Applying (18) or (19) and using (18) and  $t_2 = 2 t_1$ , we find that

$$(28) \ \tau = - \ t_1 \ / \ (\ln \ (1 - T_1 \ / \ A))$$
 (Please note that, as  $0 < (1 - T_1 \ / \ A) < 1$ ,  $\ln \ (1 - T_1 \ / \ A) < 0$ , causing  $\tau > 0$ .)

In (28)  $\tau$  is expressed in term of A.

If this is unacceptable, you can insert (27) into (28) and will find

$$(29) \tau = -t_1 / (\ln (1 - T_1 / (T_1^2 / (2 T_1 - T_2))))$$

$$= t_1 / (-\ln (1 - ((2 T_1 - T_2) / T_1)))$$

$$= t_1 / (-\ln (T_1 - 2 T_1 + T_2) / T_1)))$$

$$= t_1 / (-\ln ((T_2 - T_1) / T_1)).$$

And finally we get that

(30) 
$$\tau = t_1 / (\ln (T_1 / (T_2 - T_1)))$$

where  $\tau$  is expressed only in  $t_1$ ,  $T_1$  and  $T_2$ .

As control we can insert (27) in (19), which yields

(31) 
$$\tau = -t_2 / (\ln (1 - T_2 / (T_1^2 / (2 T_1 - T_2))))$$

$$(32) = 2 t_1 / (-\ln (1 - T_2 (2 T_1 - T_2) / T_1^2))$$

$$(33) = 2 t_1 / (-\ln (T_1^2 - 2 T_1 T_2 + T_2^2) / T_1^2))$$

(34) = 
$$2 t_1 / (-\ln (T_1 - T_2)^2 / T_1^2))$$

$$(35) = 2 t_1 / (\ln (T_1^2 / (T_1 - T_2)^2))$$

$$(36) = 2 t_1 / (2 \ln (T_1 / (T_1 - T_2)))$$

$$(37) = t_1 / (\ln (T_1 / (T_1 - T_2)))$$

Which is the same as (30). The solution is found in (27) and (28) – or more fully in (27) and (30).

Birkerød, Denmark. 27/10 2022. Torkil.

# 9.2. My own – unsuccessful – attempt to deduce a general analytic expression for P x $R_{th}$ and $\tau$ .

In section 8.2. "The transient temperature response and the thermal time constant", we saw, that the equations, which governs *the rise* and *the fall* of the internal temperature of the solar oven - as a function of the time (t), are given as:

- (1) By rising temperature:  $T(t) = T_{Amb.} + (P \times R_{th}) (1 e^{-t/\tau})$  and
- (2) By falling temperature:  $T(t) = T_{Amb.} + (P \times R_{th}) e^{-t/\tau}$ .

The first equation shows, that it will literally take "for ever" (i.e. infinitely long) for the temperature to reach its maximum. In fact, it will never reach its maximum – only approach its maximum  $T_{Final}$  - asymptotically. Thus, to be able find the maximum temperature and the rise time of the temperature based on some few measurements points of the temperature as function of the time, would be a major advantage.

#### Preconditions for the measurements:

- 1. At the time  $t_0 = 0$ , when the oven is first exposed to the sun, the internal temperature of the oven must be identical to the temperature  $T_{Amb.}$  of the surroundings.
- 2. T<sub>Amb.</sub> must remain constant during the measurements.
- 3. The first temperature in the oven  $T_1$  is measured at the time  $t_1$ .
- 4. The second temperature in the oven  $T_2$  is measured at the time  $t_2$ .
- 5. P x R<sub>th</sub>.is hereafter called A for the sake of simplicity.

My aim has been to deduce a *general solution* (i.e. a solution valid for any ratio of  $t_1$  and  $t_2$  – except for the ratios equal to: zero, one and infinite). I do not know if a general analytical solution exists to this mathematical problem with two exponential equations and two unknowns. Yet, for the case that my (SC's) attempt to solve it, could be of any use to somebody - I have included it.

The goal is get  $A = P \times R_{th}$  and  $\tau$  expressed in terms of two measurements of the interior temperature of the oven  $T_1(t_1)$  and  $T_2(t_2)$  (measured at the time  $t_1$  and  $t_2$  - after start of the irradiation). At the graph below the two measurements are represented by the two points  $(t_1, T_1)$  and  $(t_2, T_2)$ .

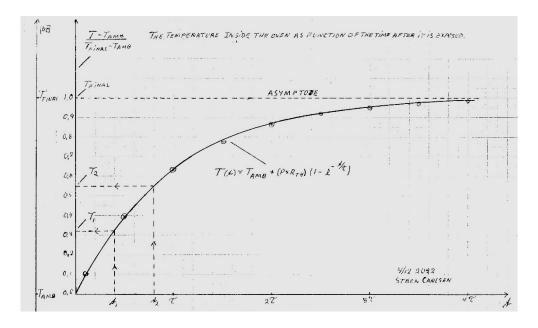


Fig. 14.

If we insert the values of the two measurements  $(t_1, T_1)$  and  $(t_2, T_2)$  into

(3) 
$$T(t) = A (1 - e^{-t/\tau})$$

we obtain two exponential equations with two unknowns:

(4) 
$$T_1 = A (1 - e^{(-t_1/\tau)})$$

(5) 
$$T_2 = A (1 - e^{(-t_2/\tau)})$$

This gives us two exponential equations with the two unknown figures:  $\tau$  and A.

(6) 
$$T_1 = A (1 - e^{(-t_1/\tau)})$$
 - and

(7) 
$$T_2 = A (1 - e^{(-t_2/\tau)}).$$

$$(6) = >$$

(8) 
$$T_1 / A = 1 - e^{(-t_1/\tau)} / A$$

(9) 
$$(T_1 / A) - 1 = -e^{(-t_1/\tau)}$$

(10) 
$$1 - (T_1 / A) = e^{(-t_1/\tau)}$$

(11) 
$$\ln (1 - (T_1 / A)) = \ln (e^{(-t_1 / \tau)})$$

(12) 
$$\ln (1 - (T_1 / A)) = -t_1 / \tau$$

$$(13) \tau = (-t_1) / (\ln (1 - (T_1 / A))) = (-t_2) / (\ln (1 - (T_2 / A)))$$

I will now try to insulate A.

$$(14) \ (-t_1) \ / \ (-t_2) \ = \left( \ln \ (1 - (T_1 \ / \ A)) \right) \ / \ (\ln \ (1 - (T_2 \ / \ A))) \quad \wedge \quad \ln \ K^X = X \ \ln \ K$$

(15) 
$$\ln (1 - (T_1 / A)) = ((t_1) / (t_2)) (\ln (1 - (T_2 / A)))$$

(16) 
$$\ln (1 - (T_1 / A)) = \ln (1 - (T_2 / A))^{((t1) / (t2))}$$

$$(17) \ 1 - (T_1 / A) = (1 - (T_2 / A))^{((t1) / (t2))}$$

$$(18) T_1 / A = 1 - (1 - (T_2 / A))^{((t1) / (t2))}$$

(19) 
$$T_1 / A = 1 - (1 - (T_2 / A))^{((t1) / (t2))}$$

(20) 
$$1 / A = (T_1)^{-1} (1 - (1 - (T_2 / A))^{((t1) / (t2))})$$

$$(21)\;A=T_1\,/\,(1-(1-(T_2\,/\,A))^{((t1)\,/\,(t2))}))$$

(22) A 
$$(1 - (1 - (T_2 / A))^{((t_1) / (t_2))}) = T_1$$

(23) A - 
$$(A - A (T_2 / A))^{((t1)/(t2))}) = T_1$$

$$(24)(T_2)^{((t_1)/(t_2))} = T_1 !!!!$$
 - And here I stalled!

## 9.3. The un-linearity of the thermal resistances as a function of the temperature.

So far, we have treated the thermal resistances and capacitances as if they were linear components. For thermal capacitances, this is almost true and generally it is a fair approximation to regard them as linear.

Opposite, for the thermal resistances. As we saw it in section 8.3. the thermal resistances are composed of three parts;

- A. The *conductive* part, which is fairly linear and
- B. An un-linear *convectional* part (with the temperature T raised to the power of 1.25; i.e. T<sup>1.25</sup>) and
- C. A radiation part, with an even more un-linear function of the temperature T

(raised to the power of four; i.e.  $T^4$ ).

Given the non-linear nature of the convection and not least the radiation, I want to have a look into recorded measurements to see how the resulting thermal resistance (between the inside of the solar oven and the ambient) change as a function of the temperature inside of the solar oven.

For this purpose, I have selected a number of data sets  $(t_1, T_1)$  covering inside temperatures of the solar oven ranging from 56°C to 103°C and calculated their respective thermal resistances.

T <sub>Final</sub>	[°C]	56	69	81	86	88	88	92	99	103
$T_{Amb.}$	[°C]	28	28	28	30	25	28	28	25	30
T <sub>Final</sub> - T <sub>Amb.</sub>	[K]	28	41	53	56	73	62	64	74	73
P	[W]	42	62	83	103	125	107	121	151	175
R <sub>Th.</sub>	[K/W]	0.67	0.66	0.63	0.54	0.58	0.58	0.53	0,49	0.42
Aug 2022	Date/Ho.	11 17-18	11 15-17	11 14-15	12 16-18	11 09-10	12 12-14	11 11-12	11 08-09	12 14-16

The values of  $R_{\text{Th}}$ , calculated based upon the subjectively estimated final temperatures - listed in order of increasing temperature of the oven.

The table shows how the value of the thermal resistance ( $R_{Th.}$  between the oven and the ambient) drops from 0.67 K/W to 0.42 K/W as the temperature of the oven rises from 56°C to 103°C. This corresponds to an average drop of 2.1 % K<sup>-1</sup>.

The plot shows limited variations of the thermal resistances - indicating that the estimations of  $T_{\text{Final}}$  seems to be fairly accurate.

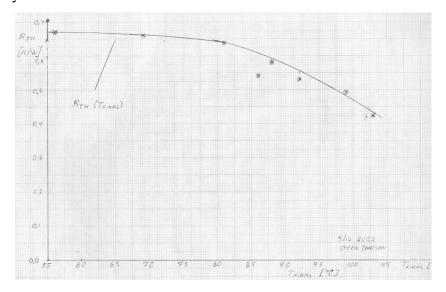


Fig. 15.

- 10. Design considerations.
- **10.1.** The thermal trade-offs of the size and shape of the solar oven. Section 8. "Thermal properties" taught us, that the irradiated power from the sun to the solar oven is proportional to A. The area of the pane in the lid of the solar-oven and
- B. The power dissipated from the solar oven to the ambient, witch (with a good approximation) is proportional to the total area of the solar oven.

That means that we should aim for the highest possible ratio between the area of the pane – and the entire area of the solar oven.

Thus, the height of the sides of the oven should be no higher, than what is absolutely needed.

Another side of the same issue is, that if two or more solar ovens are placed side by side – touching each other - the power loss through / from the side(s) of the oven, which faces a vertical side of a neighbouring oven will almost be cancelled.

Hence, an oven, which touches another oven, will achieve an increase in temperature from  $T_1$  to  $T_2$ . If we accept the approximations, that the thermal resistance is inversely proportional to the surface area - and that we can ignore the area of the pane – we have, that the ratio

- (1)  $(T_2 T_{Amb.}) / (T_1 T_{Amb.}) = A_1 / A_2 where$ 
  - $A_1$  = The entire surface area of the oven minus the area of the glass and
  - $A_2$  = The entire surface area minus the glass and minus the area of the vertical side of the oven, which touches the other oven.
- (2)  $A_1 = (2 \cdot l \cdot h + 2 \cdot w \cdot h + l \cdot w)$  and
- (3)  $A_2 = (1 \cdot h + 2 \cdot w \cdot h + 1 \cdot w).$
- (4)  $(T_2 T_{Amb.}) / (T_1 T_{Amb.}) = A_1 / A_2 = (2 \cdot l \cdot h + 2 \cdot w \cdot h + l \cdot w) / (l \cdot h + 2 \cdot w \cdot h + l \cdot w)$   $\Rightarrow (T_2 T_{Amb.}) = (T_1 T_{Amb.}) (2 \cdot l \cdot h + 2 \cdot w \cdot h + l \cdot w) / (l \cdot h + 2 \cdot w \cdot h + l \cdot w)$   $\Rightarrow T_2 = T_{Amb.} + (T_1 T_{Amb.}) (2 \cdot l \cdot h + 2 \cdot w \cdot h + l \cdot w) / (l \cdot h + 2 \cdot w \cdot h + l \cdot w).$

If we use the size of the present oven and set the temperatures  $T_{Amb}$ . = 30°C &  $T_1$  = 80°C - and The Height = 280 mm. The Width = 380 mm. And the Length = 575 mm.

```
\begin{split} &T_2 = 30^{\circ}\text{C} + (80^{\circ}\text{C} - 30^{\circ}\text{C}) \; (2 \cdot 575\text{mm} \cdot 280\text{mm} + 2 \cdot 380 \; \text{mm} \cdot 280\text{mm} + 575\text{mm} \cdot 380\text{mm}) \, / \\ &(575\text{mm} \cdot 280\text{mm} + 2 \cdot 380\text{mm} \cdot 280\text{mm} + 575\text{mm} \cdot 380\text{mm}) \\ &= 30^{\circ}\text{C} + \\ &50^{\circ}\text{C} \; (322000\text{mm}^2 + 212800\text{mm}^2 + 218500\text{mm}^2) \, / \; (161000\text{mm}^2 + 212800\text{mm}^2 + 218500\text{mm}^2) \\ &= 30^{\circ}\text{C} + 50^{\circ}\text{C} \; (753300 \; \text{mm}^2) \, / \; (592300 \; \text{mm}^2) = 30^{\circ}\text{C} + 50^{\circ}\text{C} \cdot 1.27 \end{split}
```

This calculation is not exact as it does not take into account the un-linearity of the thermal resistances. Still it provides a clue about the magnitude of the potential improvement.

If placing three or more solar ovens side by side it is possible to obtain a still higher increase(s) of the internal temperature(s) of the solar oven(s) in the middle (- all though, the subsequent increase will be smaller than the first increase).

10.2. Insulation of the air gap in the solar oven. When I started the solar oven project, I believed that a space of 10 mm air between the inner- and the outer boxes would provide for an adequate thermal insulation for the oven. The test of the oven taught me that two surfaces that are near to one another causes a significant increase of the convection losses. Meanwhile, I have also learned that for buildings a non-insulated cavity walls will typically exhibit a thermal U value of approximately  $10 \text{ W/m}^2/\text{K}$  (compared to  $0.2 \text{ W/m}^2/\text{K}$  for an insulated roof). This have led me to the conclusion, that the 10 mm air gap between the inner- and outer box should be filled with some kind of insulating material.

**10.3. Insulation materials.** The "hay box" owes its name to hay's property as an excellent thermal insulator. I assume that dry grass is almost as good as hay!

My search for suitable insulation materials, which would be generally available in most geographical regions, pointed me in the direction of wool from sheep as one of the best insulators. With a limited knowledge of wool, I approached three wool experts:

A. one was employed at the Technological Institute of Denmark,

B. one was manufacturing high temperature protective gloves and suits for fire brigades - and

C. one was running a wholesale company selling different grades of wool.

They all generously shared their knowledge with me. They convinced me that unwashed wool with lanolin will have long lifetime and that it would withstand temperatures up to and beyond the boiling point. The lady C, who runs the wool company - in her kindness - even sent me a big bag with these samples of wool (fig. 16).



Cooking without cleaning can be a threat to health.

Fig. 16.

Thus, to keep the inner box tidy, it must be simple to remove it from- and refit it into the outer box. If the wool must be squeezed into the gap between the two boxes it will hamper the removal of- and the refitting of the inner box into the bigger outer box – and thus, the cleaning.

Hence, to ease the insertion of the insulating material into- and removal from the air gap (may it be wool, hay or other insulating materials), I will suggest to make two rectangular shaped "duvet covers" A and B for the wool. The two covers are placed between the two boxes. The purpose of the two covers is to distribute the wool evenly / to spread it out, inside the two covers, and to protect the wool from dirt.

The A-cover should have a size and shape identical to the size of the outside bottom of the inner box.

The B-cover should have a *width* identical to the height of the inner box.

The B-cover should have a *length* of:

- a. Twice the width of the air gap. +
- b. Plus, twice the length and the width of the outside of the inner box. +
- c. Plus, twice the width of the outside of the inner box.

The A-cover is placed first (as a horizontal mattress) in the bottom the outer box.

The B-cover is inserted vertically in the space between the inner and the outer box.

Do not stuff more wool into the two covers, than their thickness still allows for the covers to easily be removed and refitted into the air gap (i.e. the space between the two boxes).

To keep the wool in place in the covers, you can apply a number of stitches evenly divided over the surface of the cover.

If you cannot find wool, use hay, dry grass or even seaweed. The important issue is to use a material, which has very low thermal conductivity (which often means that it consists of a high percentage of air), and which has a physical structure, which presents a high obstruction to air movements.

Remember that insulation material must be kept dry in order to maintain its purpose -i.e. to insulate. If sauces, food or water, which is spilled over from pots or pans contaminates the insulation between the two boxes, it will ruin the insulation.

Hence, it is recommended to seal off the space / air gap. A cheap, simple and efficient solution is a slightly inflated bicycle tube. A bicycle tube of a suitable length will with its elasticity allow it (just as a rubber band) to be fitted around the upper rim of the inner box. If a bicycle tube cannot be found, use a piece of rolled fabric or a rope with a suitable diameter and length.

There are many industrially manufactured insulating materials, which are good. But they are often expensive and generally not available in rural areas. Even so, I will mention two of the best and most prevailing, which are mineral wool, glass wool and fibreglass. All three have good insulation values and are non-flammable. Some people claim that they are both environmentally friendly – I disagree on that! Both materials can cause skin allergy.

But they have other good properties. Mineral wool does not melt, nor does it support combustion. Unfortunately, it is fluffy and disintegrates. Fiberglass, which is more "solid" does not absorb water. My advice is: If you stumble over any of these insulation materials – consider if you can / or will use them; but do not take any detour to get hold of them.

## 11. Dressing up the solar oven in coat and skirt.

What is a life without a good laughter? Here comes a suggestion to have some fun by dressing up your solar oven. When people freeze, they put on warm clothes. A solar oven will always freeze and long for a higher temperature. What would be more caring, than dressing up your oven in warm clothes? –just remember to leave the pane uncovered, for the sun to peep into the oven.

An excellent dress for a solar oven would be a big sized Scandinavian winter coat, with a neckline large enough to encircle the glass in the pane / the upper side of the oven - for the sun to get in. Realizing that a Scandinavian winter coat is a rare commodity in Africa, a jacket or even a big bag will be much better than nothing. If the jacket or the coat, which you intend to use, have sleeves (or other openings / holes) tie a knot on each of the sleeves, to prevent draught to cool down the oven.

The European climate is cool and for centuries women's fashion has been long skirts; right down to the ground. The long skirts were not only to look fancy, it was just as much to prevent draught and to keep warm. The same needs apply to solar ovens! They freeze and want to be good looking. Thus, when you tailor, a coat, a jacket, a skirt – or whatever fabric for the oven, try to make it long enough to reach down to the ground – i.e. long enough to prevent draught to get underneath the inclined solar oven, and cause it to freeze.

**11.1.** The impact of a coat on the temperature of the oven. The heat dissipated from an idling adult person is in the order of 100 W. This power is comparable to the 147 W of power, which we calculated is irradiated from the sun into the solar oven. The internal temperature of the human body is 37°C, and the external temperature of my stomach is 31°C. Without coat I will feel comfortable down to an ambient temperature of approximate 20 - 25°C – while with the coat I will be comfortable down to an ambient temperature of approximate 5°C – maybe less. Thus, in all my optimism, a coarse estimate leads to a possible increase in the operating temperature for the solar oven of somewhere in the range of 10°C - 15°C.

Wind and draught will cool of the solar oven, thus, try to find- or establish- a shelter, where it can be placed. A half-brick wall or a stone fence will serve the purpose and add to the insulation. If you have access to palm- / banana-leaves place- / lean them up towards the vertical sides of the oven.

Whatever insulation material is used, it must be protected against wind and water by some sort of wind- and water proof material. A fur's ability to shield against both precipitation and wind makes it a good means to insulate and protect the oven and its insulation. The wool experts I have spoken to advocated for an unwashed sheep fur, which provide both insulation and water proof protection.

### 12. Pots and pans used in the East African kitchen.

It would be nice if the space and dimensions of the solar oven would be able to accommodate any sizes and shapes of the user's typical pots and pans – and their handles! But in section 10.1. "The thermal trade-offs of the size and shape of the solar oven." I found, that for the sake of the operating temperature the height of the sides of the oven should be no higher, than what is absolutely required. Hence, the depth of the inside of the oven should be limited to the height of the highest pot – or to the height of its handle - plus the headroom required for the inclination of the oven when it is adjusted to the altitude of the sun.

It is my experience, that many African meals require two or more pots or pans, and that many users of solar ovens therefore would want their ovens to be able to accommodate at least two pots or pans.

Initially I expected that most handles would stick out horizontally from their pot / pan. (I later learned that Zambian handles sticks out in an angle of 35°-45° relative to horizontal.)

Hence, I have shaped the bottom of the solar oven as a rectangle, which is just big enough for the circular parts of the pots or pans to be placed in opposite ends (corners) of the oven. The handles will have to point towards opposite ends of the oven and the handles will be more or less parallel to each other see fig. 17.

Looking for information on typical sizes of East African pots and pans – and the sizes of their handles, I turned to my friend Jesper Berggreen. Jesper and his sister Line have during their childhood, adolescence and adult life spent several long periods in Zambia and Kenya. Jesper and Line provided me with a number of pictures (taken in 2014 and 2016), which show Zambian pots and / or pans (fig. 18-20).



Fig. 17.



Fig. 18.

Using a slide calliper and photogrammetry I found the diameter of the pan and the length of the handle. Knowing that Line's right arm is 325 mm from the outer elbow to the thumb base, I found the diameter of the pan to be  $\emptyset = 36.73$  mm x (325.0 / 60.51) = 197.3 mm  $\emptyset$ . Likewise I find the length of the handle of the pan to be 27.32 mm x (325.0 / 60.51) = 147 mm.

Scrutinizing the photos to the right, I discovered that all the pots had handles, which were inclined in the order of 35°- 45° relative to horizontal. This is unfortunate, as it increases the required height of the solar oven, which in turn would lower its operating temperature.

The inclination of the handles also increases the risk for the user to be burned by the steam from the pot. The only reason for the inclination seems to be the manufactures' wish to reduce the tensile strength on the rivets in the joints between the steel handles and the aluminium pots.



Fig. 19.

If you want decrease the inclination of your pot handle you can cut a chink in the underside of the handle with a hacksaw. Cut through half the thickness of the handle and cut near to the side of the pot. Adapt the width of the chink so that the sides of the chink will meet / join when the handle has been bent downward to its desired inclination. This will also reduce the risk of being scorched by the steam from the pot.

The joint between the two edges of the chink will now take up the entire torque in the handle - i.e. the handle will maintain its strength.



Fig. 20.

If you so wish, you can ask a black smith to give the joint a welding spot.

I am embarrassed to admit that I do not remember anything about kitchen utensils from my own visit to Uganda in 2004. The solar oven project started primary aiming to alleviate the consumption of fire wood in the refugee camps in Northern Uganda and Northern Kenya. After I learned about Zambian pot handles, I wondered where to find information on the length and inclination of pot handles in Uganda and Kenya.

In 2009 the Ugandan Ambassador to Denmark Joseph Tomusange invited me for tea. Hence, I got the idea to call the Ugandan Embassy and ask them for information on the length and inclination of pot handles in Uganda.

The lady at the Embassy, to whom I talked to was bit astonished to get that question. She kindly informed me that Ugandan pots do not have any handles. Instead of handles, at the top of the Ugandan pots, they have a horizontal outward bent rim. When moving pots in Uganda, you use banana leaves as pot holders. The Ugandan lady stated that the Embassy had high interest in the solar oven project, and invited me to bring the oven and this report along.

Based on the acquired information on East African kitchen utensils and their handles (which is a completely new subject to me) I have made up my mind not to sacrifice the operating temperature on the altar of the internal depth; and decided on 210 mm. as a suitable trade-off between the factors:

- A. Operating temperature.
- B. Height of pot.
- C. The inclination angle of the oven.
- D. The possible inclination of pot handles in Kenya and Zambia and elsewhere.

And now from the issue of the height of the oven - to the horizontal shape and size of the oven. Many East African pots will have handles, which calls for a rectangular format of the solar oven. A rectangular format will generally allow for two pots - side by side – providing either for two meals to be cooked simultaneously or - just one pot and a number of water bottles to produce sterilized drinking water. From Jesper Berggreen's sister Line I have learnt that Zambian pots and pans often have diameters in the order of 200 - 250 mm.

To estimate the required content of the pots - and thus the oven - I have asked around about the number of persons in groups- or families of refugees, who arrive in the refugee camps. The number varies a lot, but generally, it is within the range between three and eight persons, with an estimated number on four to five persons as the mean value.

Danish physicians have various guidelines for the required energy intake per day for humans. One guideline is 30 kJ per kg body mass by normal weight and activity level. For a person with a weight of 70 kg the daily need of energy amounts to 2.100 kJ. If an energy intake of 2.100 kJ should be provided by rice only, (which has an energy content of 1500 kJ / 100 g), it would require 2.100 kJ / (1.500 kJ / 100 g) = 1.4 x 100 g = 140 g rice per day.

900 g of dry rice correspond to a volume of 1.0 litre. Thus, 140 g of dry rice correspond to 0.173 litre, which when cooked will swell to approximate 0.35 litre. One Calorie = 4.184 Joule. (The fact that this calculation only comprise rice, does not imply any justification of the fact that many refugees live of an unbalanced diet.)

If a family has five members, the water required amounts to 1.75 litre. With a solar intensity of 1000 W/m2, a solar-oven with an aperture of 0.30 m x 0.49 m can provide a power P of 147W. To heat 5 servings of 0.35 litre (= 1.75 litre) from e.g.  $25^{\circ}$ C to  $75^{\circ}$ C with a heating power of 147 W (and an efficiency of 100%) takes

```
t = C_{Th} / P = (1.75 \text{ litre x } 1.0 \text{ kg/litre x } 4184 \text{ Joule/(kg x K) x } (75^{\circ}\text{C} - 25^{\circ}\text{C}) / 147 \text{ W} = 2490 \text{ s} = 41\frac{1}{2} \text{ minutes}.
```

This heating time is under perfect conditions - i.e. an infinite high thermal resistance between the interior of the solar-oven and the ambient - and a solar-oven with no thermal capacitance.

## 13. Using the solar oven as a hay box.

No matter whether you use the oven as a solar oven or as a hay box, placing a lid on top of pots and pans will speed up the cooking process and save fuel. Use of lids will also reduce the humidity in the oven and thereby prolong the lifetime of its wood. This is because: Whenever water or liquid evaporates and turns into steam, that process will convert and store energy into- and as steam. Without a lid on top of the pot, the steam on top of the food or water will "steam away" leaving room over the surface and allow space for new steam to be generated – thereby causing more energy to be used for (the un-desired) evaporation.

When an oven is used as a hay box, the cooking process is started by heating the food to a high temperature (typically just below 100°C). The source of the initial heating could be either the sun or a combustion of fossil fuels, burning wood - or whatever.

When the food or liquid has attained the desired (or obtainable) temperature, it is placed in the hay box. A good insulation of the oven will allow a prolonged and slow decrease of the temperature of the food / liquid. When the oven goes from working as a solar oven to operate as a hay box, I can recommend to cover it with blankets or whatever is available to keep it warm. During the decrease in temperature the cooking will continue - without further external heat supply. Try to resist the temptation to open the pane before the food is ready. If you do so the heat in the oven will escape.

The toll of the decreasing temperature is the prolonged cooking time (Please recall section 4. "Svante Arrhenius' law.") Still, using a good insulation of the hay box will save a significant amount of fossil fuel (my guess would be 30 - 70%) – depending on the oven, the meal and the amount of food / liquid to be processed. Bear in mind, that to reduce risk of bacterial contamination, the food should be served and eaten less than an hour after the temperature has sunk below  $60^{\circ}$ C.

When using the oven as hay box, very hot pots and pans will often be placed in the oven. In order to protect the bottom of the oven (generally wood), I recommend to make some sort of insulation, such that the pots and pans will not excessively heat the wood at bottom of the oven. Some sheet of metal (e.g. steel, aluminium copper) placed on top of a few pebbles or a tiny pieces of clay will do the job.

Keep in mind, that the more the metal and/or the insulation weights - compared to the pot with food/ liquid – the more will the metal and/ or the insulation absorb of the initial heat / thermal energy of the pot (with its content) – and the longer will it take to cook the food. Thus, mind the weighs!

If food is kept within the thermal danger zone (i.e. 5-60°C) for longer than one hour - there is a risk of bacterial growth. The two most common ways to avoid this risk is to:

A. ensure that the insulation of the hay box is so good that the preparation / cooking of the food is accomplished well before the temperature in the hay box has fallen below the 60°C -

B. and in case the temperature has been below 60°C -/ then to reheat the food (to minimum 72°C – and preferably to the boiling point) before it is served.

In either case, it does make sense to keep the temperature of the oven, as high as possible for as long as possible by covering the oven with coats / shirts / fabrics / furs etc. – i.e. anything, which will add further insulation to the oven – as soon as the sun can no longer rise or maintain the temperature of the oven.

## 14. Notes, mistakes made, lessons learned, recommendations, tasks and rectifications.

- 1. My first mistake was to not to read enough about other people's experiences with solar ovens before I started the project.
- 2. It was a mistake was to use heavy 10 and 15 mm chipboards with an unsuitable high thermal capacity rather than ordinary wooden boards. The weight of my chipboard solar oven turned out to be 18 kg. It would be unfair to ask anybody to carry that weight every day.
- 3. For future ovens, I suggest to use plain wooden boards preferably with tongue and groove. To get a lighter solar oven, I suggest to use a board thickness between 8 12 mm. The lower limit of 8 mm is chosen to prevent the boards from splitting when the screws are applied. I recommend to pilot-drill a hole with a diameter 0,5 mm less than the diameter at the bottom of the thread of the screws a slide calliper is useful when measuring this diameter.

  Nb. The thermal expansion coefficients for plain wood parallel to the grain is: 30 x 10<sup>-7</sup> K<sup>-1</sup>; and perpendicular to the grain it is: 300 x 10<sup>-7</sup> K<sup>-1</sup>. Hence, if the boards used for the longer sides are horizontal, the boards for the ends must not be vertical they must also be horizontal. Otherwise the oven might crack.
- 4. The thermal expansion coefficients are for: glass 6.7  $10^{-7}$  K<sup>-1</sup>; & wood parallel to grain:  $30 \times 10^{-7}$  K<sup>-1</sup>. The length L of the glass = 0.53 m. Increasing the temperature from 0 100 °C = 100 K; causes the length of the glass to exceed the length of the wood in the frame by:  $\Delta L = 100$  K \* 0.53 m \* (67 30) x  $10^{-7}$  K<sup>-1</sup> = 0.196 mm. Hence, I suggest to design for a play = 1 mm between the glass and the frame.
- 5. Draught will ruin the insulation of the oven. Hence, do make all joints as airtight as possible. On wooden boards, paint, glue or egg yolk can be used to seal the cracks. If you have not been able to find boards with tongues and grooves use some kind of sealing material at the surfaces were the boards join. Paint, used as seal to the sealing might prolong its life time.
- 6. Knowing how difficult it will be to find parts and tools in a refugee camp, I wanted to try to build a prototype of a low technology solar oven using mainly recycled or left over materials. I used discharged chipboard from a wardrobe left as garbage in the street. The glass sheets were left over etc. Lot of money to be saved by using recycled materials. Thus, if you wish to build a low budget solar oven or a hay box start early collecting tools and materials.
- 7. Glass cutters are available in two versions: A. Diamond based cutters and B. Cutters based on hardened steel rollers. Some shops recommend to use special glass cutting oils I have always used alcohol and kerosene. When cutting a straight line, apply two clamps very, very careful! at each end of a ruler, so that you do not crush or crack the glass. Apply the "cutting oil" to the glass surface to be cut. Move the glass cutter along the ruler in **one** single stroke reaching from edge to edge, while pressing it towards the glass with a force of somewhere between 10 and 50 Newton (corresponding to 1 5 kg). When you have applied a sufficient pressure you will start to hear the sound of thousands of tiny crushes of the glass under the steel roller of the glass cutter. When you crack the glass, place the line to be cracked 2 cm inside a (straight) edge of a table and apply a vertical pressure to the glass at both sides of the line. To avoid the glass cutting try to find a pane, which is already fitted with a double-glazed window or even better a sealed unit and then build / adapt the solar oven to fit around that pane.

- Beware! Glass cutting is dangerous and the edge of the glass is sharper than any knife. The present solar oven has taken its toll of my blood!
- 7 Prevent your solar oven from being stolen, by keeping it in your house at night.
- When assembling a solar oven use screws rather than nails. Screws allow simple dismantling and repair. A still much stronger and more airtight construction can be made by assembling the wooden parts with both glue and screws. If possible, use torque screws. Torque screws almost never slip. even if / when they corrode.
- When you proudly have finished your new solar oven, but do not have access to a thermometer, try on a hot day with clear sky and the oven aligned directly towards the sun to place a bit of water on a small plate, and see if it starts to boil. If it boils, you have done a really good job. If it does not boil, you *might* also have done a really good job!
- 10 If you, want to measure the thermal resistance from the inside of the oven to ambient, try to find a high number (e.g. 0-30) of low power (2-10 W) incandescent bulbs /lamps. You can use lamps of different power. Mount the lamp sockets evenly distributed on a board of the same size as the bottom of the inner box, so that you easily can adjust the total power by connecting and disconnecting more or less lamps. Remember to spread the heat generation as evenly as possible to avoid the glass to crack.
- By increasing temperature in the interior of the oven the thermal resistance from the inside of the oven to ambient, will drop. If the measured temperatures cover a large temperature range, it can distort the exponential step response curve and offset the measurements of  $(t_1, T_1)$  and  $(t_2, T_2)$ .
- 12 A suitable coat should be (re)designed for the oven to increase its insulation.
- I should have measured and compared the mechanical strength of:A. Chip board, B. plywood and C. Plain lumber (parallel and perpendicular to the grains),and review alternative materials with an open mind.
- 14 For a Mark II prototype, to review (if needed) the size and shape of the bottom of the inner box in respect to accommodate prevailing sizes of pots, pans and their handles (if any).
- 15 For a Mark II prototype, (in view of 13 & 14) to reconsider the width of the insulation gap between the boxes.
- 16 To test various locally available exterior insulations materials like straw, grass and hay to be piled up towards the outside of the oven and find a suitable waterproof cover for it.
- 17 To test whether plastic in the pane could be a suitable alternative to glass, to double glazed and to sealed units.
- 18 The glass of the solar oven must be cleaned regularly to prevent dust and grease to lower the intensity of the sun and hence the temperature in the oven.
- 19 And so on.

### 15. Other cooking technologies.

There are several other different technological approaches to the use of solar power for cooking.

A. One common technology is the parabolic concentration of sunrays, which – compared to the present solar oven - provide much higher operating temperatures, at the expense of a more complex technology - and cost.

B. Another technology is electric solar panels with batteries and special cooking plates and special dual walled pots. Its great advantage is the ability to cook independent of the sun. Yet, both its technology and the price tag seem prohibitively high. Moreover, no solar panel comes with an efficiency higher than 25%. Thus, any solar panel technology will lose 75% of the available power up front. Finally, the lifetime and the chemistry of the battery will cause environmental problems.

Hence, with the present perspective, to my opinion, priority should be given to simple and cheap solutions, which, as far as possible, can be manufactured locally by the end users.

I would like to convey my whole hearted thanks to the following people and institutions

## 16. Thanks given.

**Thomas** 

Torkil.

listed alphabetically – for their support and contribution to this project:
Cathrin
Gunnar
Henrik
Jesper
Klaus
Line
Per Bo
Stig
The Ugandan Embassy in Copenhagen
The Danish Technological Institute

#### 17. Conclusion.

A prototype of a cavity walled double-glazed solar oven with room for two pots or pans has been designed and build. An operating temperature of 83°C has been achieved, which is 8°C higher than the estimated temperature of the Ladakhi solar ovens. Based on calculations, test results and the lessons learned, I believe, that the temperature of a subsequent prototype can be increased by another 5°C -15°C. The design, the technology, the choice of materials and required tools have been aimed to be adapted to the conditions in refugee camps. I have tried to use either recycled or cheap lightweight materials. I have strived to make a design, which can be made by a skilled handyman.

During this project many thoughts and considerations have gone through my mind and associated ideas have evolved. The extensive issues have got each their own section and are listed in section "1. Contents.". The rest (not necessarily inferior) are grouped together in section "14. Notes…"

From the early 20<sup>th</sup> century hay boxes have been used extensively to reduce the consumption of fuel. It seems obvious to merge the functions of solar ovens and of hay boxes, which would imply fuel / firewood savings even during the rainy seasons. Another additional purpose for the solar ovens, is to sterilise contaminated water to reduce the high mortality caused by muddy drinking water.

In any design / in any activity - a feedback from the end user is crucial. Please help me improve by providing me with feedbacks, may it be suggestions, questions, experiences or whatever.

There is however, a deadline for the feedback as I suffer from a number of incurable diseases. One of them, a progressing Parkinson's disease - makes it ever more difficult to use my hands, walk, work, etc. The trembling hands have reduced my writing speed to 10-20%, thus I prefer to talk. I do use WhatsApp. Hence, I cannot say if I have one, two or five years left. It is my hope with this report to reach out to organisations and individuals, who have an interest to use the report and hopefully to take my work forward.

It is my wish, that this report shall be accessible free of charge to everyone on the internet. I hope and imagine that this report could come to use in areas as: Africa, Asia and South America, where I hope the information can spread through universities. Unfortunately, my command of French – leave alone Spanish - is insufficient to produce a decent translation. I would be obliged if someone would assist me in the dissemination of this information. I have no economic interests in this project.

Aarhus, Denmark December 16<sup>th</sup> 2022.

Steen Carlsen

## Appendix; comprising test results and plots of temperature measurements.

This appendix comprises the thermal measurements taken from August 2022 – partly indoor in my home, partly outdoor in my allotment. I must confess, that the later measurements - by accident - were taken on some of the warmest summer days (25-30°C) with a cloudless bright blue sky. Such weather conditions are rare for Denmark but common in tropic and sub-tropic climates.

The thermal measurements (i.e. the temperatures inside the oven, on the external surfaces of the oven and the ambient) were taken in order to find the thermal resistances between various parts of the oven and the ambient.

The first measurements were taken in my home with the solar oven placed on my dinner table. The heating source was a sub-section of the heating element in an old cooking plate, supplied through a vario-transformer. This allowed the power, which was dissipated in the oven, to be adjusted over a range of approximately 0 - 250W (which was more than the 150W, which it turned out to be thr required power to reach approximate  $97^{\circ}C$ ).

The thermometers were a Fluke 189 multimeter with a resolution of  $0.1^{\circ}$ C; range -200 to +1350°C, - and a Brymen BM837 multimeter with a resolution of  $1^{\circ}$ C; range >> 0 to +100°C.

The purpose of the indoor measurements was to generate data from a reproducible setup. This would not have been possible with an outdoor set up, where variations in wind, clouds and ambient temperatures would be too disturbing and uncontrollable factors. During the indoor measurements the ambient temperature was maintained by the thermostat at the central heating system.

The power dissipated within the oven, was supplied from the 230 V AC mains, adjusted by the variotransformer and evenly distributed (i.e. stirred around) by a 24V computer fan, which was supplied from 0-30V/2A variable power supply.

The subsequent outdoor test was to verify compliance between thermal resistances found by the indoor measurements, the estimated intensity of the sun, the adjusted inclination of the oven, and the recorded (outdoor) temperature inside the oven. As seen in section "8.6.1 Check of the balance in the account of: heat flows, thermal resistances and temperatures." the results complied well given the expected accuracy of the measurements.

11/8 2022. Measurements taken in my home (with a thermostatically controlled ambient temperature).

Hour/Min.	$T_{Oven}$	$T_{Amb.}$	$P_{\text{Total}}$	$(T_{Oven} - T_{Amb.}) (T_{Final} - T_{Amb.})$	$R_{Th}$	τ	Notes
7,30	24.6	24.8	0	-0.2			
8,42	87.4	25.0	195.6	62.4			
8,47	89.1	25.0	195.6	64.1			
8,53	91.6	25.0	195.6	66.6			
8,54	92.3	25.0	150.6	67.3			
8,57	93.1	25.0	150.6	68.1			

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11/8 2022. Measurements taken in my home (with a thermostatically controlled ambient temperature).

Time	$T_{\text{Oven}}$	$T_{Amb.}$	$P_{\text{Total}}$	(T <sub>Oven</sub> - T <sub>Amb.</sub> )	(T <sub>Final</sub> - T <sub>Am</sub>	<sub>nb.</sub> ) R <sub>Th</sub>	Notes
Hour/Min.	°C	°C	W	K	K	K/W	
8,58	92.8	25.0	150.6	67.8			
9,00	93.2	25.0	150.6	68.2	70	2.15	
9,04	93.2	25.0	121.6	68.2			
9,26	88.8	25.0	125.1	63.8			Top side of glass is 30°C for T <sub>Oven</sub> = 90.1°C

Up to this point,  $P_{Total}$  was wrongly calculated as  $P_{Total} = P_{Fan} + I^2 R_{El}$  – disregarded that R rose approx. 10% from 250 $\Omega$  to 275 $\Omega$ .

From here and on-ward, PTotal has been calculated as  $P_{Total} = P_{Fan} + I \times V$ .

11/8 2022. Measurements taken in my home with a thermostatically controlled ambient temperature.

Time	$T_{\text{Oven}}$	$T_{Amb.}$	$P_{Total}$	(T <sub>Oven</sub> - T <sub>Amb.</sub> ) (T	Final- T <sub>Amb.</sub> )	$R_{Th}$	Notes
Hour/Min.	°C	°C	W	K	K	K/W	
10.11	91.9	25.0	132.8	66.9			
10.37	86.3	25.0	117.5	61.3			
11.03	88.7	28	120.4	65.7			Temp. of outer edge of oven = 43°C.
11.54	90.7	26	121.2	64.3	90	1.88	Temp. of outer edge of oven = 41.5°C.
12.27	91.7	28	98.4	63.7			Temp. of outer edge of oven = 42°C.
13.21	87.7	28	98.4	59.7			
13.24	87.5	28	98.4	59.5			
13.54	87.0	26	103.4	61.0			
14.09	87.0	28	83.2	59.0	86	1.72	
14.57	81.9	28	82.3	54.9			
15.00	81.3	28	61.5	53.3			
16.30	71.6	28.5	61.5	43.1			
17.00	70.1	28.5	45.6	41.6	70	1.48	
17.48	63.8	28	41,7	35.2	58	1.39	

12/8 2022. Measurements taken in my home (with a thermostatically controlled ambient temperature).

Time	$T_{\text{Oven}}$	$T_{\text{Amb.}}$	$P_{\text{Total}}$	(T <sub>Oven</sub> - T <sub>Amb.</sub> ) (T	Final- T <sub>Amb.</sub> )	$R_{Th}$	Notes
Hour/Min.	°C	°C	W	K	K	K/W	
10.15	25.0	25	119.7	0.0			Fan power reduced to 2.1 W.
10.35	37.0	25	119.7	12.0			
11.55	68.7	26.5	107.3	42.2			
11.58	69.2	26.5	107.3	42.7			
12.38	75.5	27.0	106.1	47.5			
13.54	81.4	29.0	107.5	52.4			
14.04	82.1	29.0	117.4.	53.1	88	1.82	
14.10	82.3	29.0	155.0	53.3			
14.19	84.5	29.0	136.5	55.5			
14.23	85.3	29.0	175.3	56.3			
15.33	98.2	30.0	175.3	68.2			
15.36	98.5	30	180.6	68.5			
15.38	98.7	30	180.6	68.7			
15.40	98.8	30	160.4	68.8			
15.43	99.0	30	160.4	69.0	99	2.32	
15.50	98.7	30	144.8	68.7			
16.32	97.0	30	129.3	67.0			
16.36	96.9	30	114.1	66.9			
17.05	92.3	30	102.8	62.3			
17.20	91.0	30	103.1	61.0			
17.38	90.0	29	102.1	61.0	88	1.73	Upper outer edge of oven = 42°C.
							Cooling down
21.35	26.5	26.5	82.46	0.0			
22.19	51.5	26.5	84.93	25.0			
22.23	52.9	26.5	117.8	26.4			
23.15	72.7	26.5	104.9	46.2			
23.57	80.9	26.5	121.0	54.4	90 1	L.91 When	the oven was wrapped in a towel, T <sub>Oven</sub> rose to 81.4°C.

13/8 2022 Measurements taken in my home (with a thermostatically controlled ambient temperature).

Time	$T_{Oven}$	T <sub>Amb.</sub>	$P_{Total}$	(T <sub>Oven</sub> - T <sub>Aml</sub>	b.) (T <sub>Final</sub> - T <sub>Amb.</sub> )	$R_{Th}$	Notes
Hour/Min.	°C	°C	W	K	К	K/W	
0.15	83.8	26.5	107.8	57.3	The hot plate	in the	oven was switched off for the night.
5.55	30.2	27	121.3	3.2			
8.33	84.8	28	105.4	56.8			
8.38	85.8	28	104.6	57.8			
8.51	87.4	28	104.8	59.4			
8.58	87.8	28	104.4	59.8			
9.22	89.4	28	104.4	61.4			
9.52	91.0	28	104.4	63.0			
9.54	91.0	28	0	63.0			
10.18	76.8	28	0	58.8			
11.21	54.8	29	0	25.8			
11.42	50.0	28	0	22.0			
11.46	49.6	28	0	21.6			
12.06	46.1	28	0	18.1			

Packs up and goes to the garden to continue with open-air test.

13/8 2022 Outdoor; Weather: cloud free sky and 29°C.

Time	$T_{Oven}$	$T_{\text{Amb.}}$	$P_{\text{Total}}$	(T <sub>Oven</sub> - T <sub>Amb.</sub> ) (T <sub>Fir</sub>	<sub>ial</sub> - T <sub>Amb</sub>	.) R <sub>Th</sub>	Notes
Hour/Min.	°C	°C	W	K	K	K/W	
13.11	40.3	28	0	12.3			
13.23.	52.6	28	0	24.6			
14.17	71.6	29	0	42.6			
14.46	82.7	29	0	53.7			
15.36	76.6	29	0	47.6			
16.16	78.5	29	0	49.5			
16.40	79.6	29	0	50.6			
16.45	80.1	29	0	51.1			
17.21	78.7	29	0	48.7			

1/9 2022. Change lower glass in the pane from a thickness of 5 mm to a thickness of 4 mm.

Cover the inner sides of the inner box with aluminium foil.

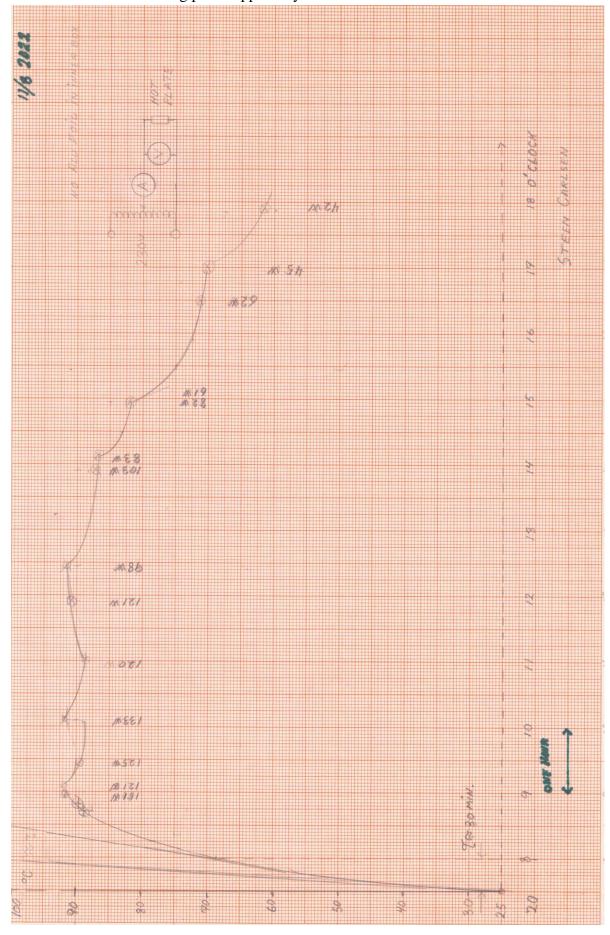
# 2/9 2022 Weather: cloud free sky and 29°C.

Time	$T_{Oven}$	$T_{Amb.}$	$\mathbf{P}_{\text{Total}}$	(T <sub>Oven</sub> - T <sub>Amb.</sub> ) (T <sub>Final</sub>	I- T <sub>Amb</sub>	.) R <sub>Th</sub>	Notes
Hour/Min.	°C	°C	W	K	K	K/W	
15.30	75.0	21	0	54.0			
16.10.	77.9	21	0	56.9			
16.40.	79.2	21	0	58.2			
17.00	80.4	20	0	60.4			
17.35	75.5	20	0	55.5			
19.00	42.7	18	0	24.7			

## 3/9 2022. Weather: clouded.

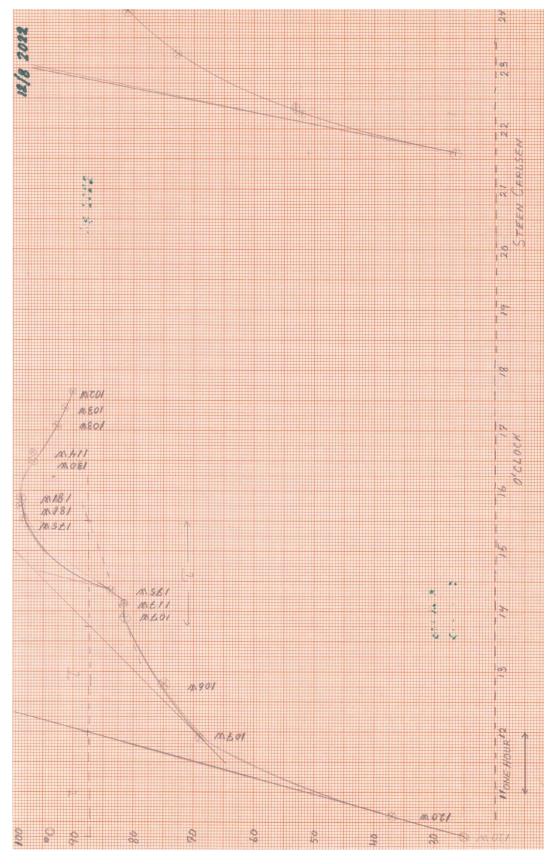
Time	$T_{Oven}$	$T_{Amb.}$	$P_{Total}$	(T <sub>Oven</sub> - T <sub>Amb.</sub> )	(T <sub>Final</sub> - T <sub>Amb.</sub> )	$R_{Th}$	Notes
Hour/Min.	°C	°C	W	К	K	K/W	
09.50	19.5	21.1.	0	2.4			
11.06	20	28.3	0	!			Hopeless weather

Indoor. Heat source: Cooking plate supplied by a vario transformer. Plot 1 Steen Carlsen 11/8 2022

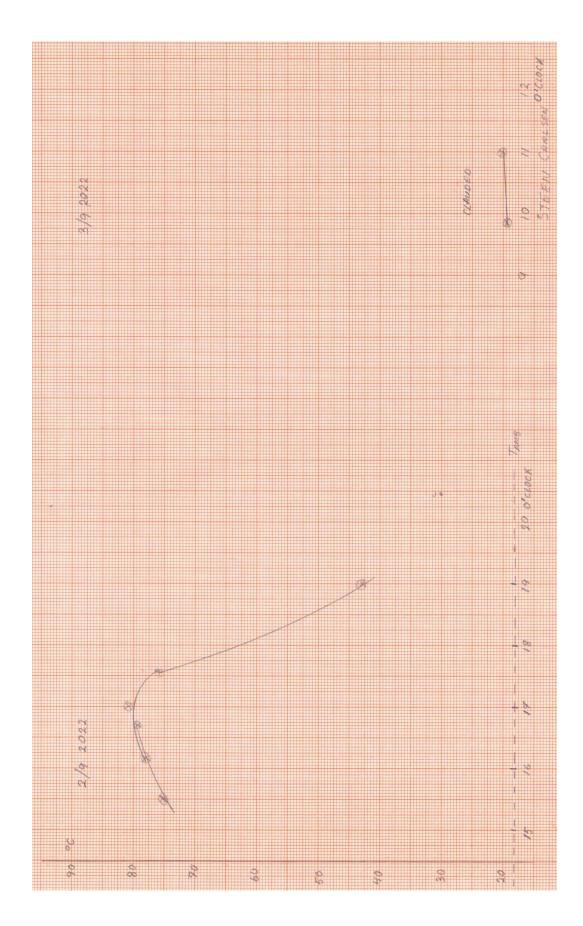


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Indoor. Heat source: Cooking plate supplied by a vario transformer. Plot 2 Steen Carlsen 12/8 2022



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